ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

OPTIMIZING REPLICATED CENTRAL COMPOSITE DESIGNS WITH FRACTIONAL FACTORIAL COMPONENTS: A STUDY ON DESIGN EFFICIENCY

CHARLES ANTHONY NSE

Department of Mathematics and Statistics, University of Port Harcourt, Nigeria.

Abstract: Efficiency and optimal properties of four varieties of Central Composite Design, namely, SCCD, RCCD, OCCD and FCCD and having r_f replicates of the Fractional Factorial portion, r_α replicates of the axial portion and r_c replicates of the center portion are studied in four to six design variables. Optimal combination,$[r_f{:}\ r_\alpha{:}\ r_c]$ of design points associated with the three portions of each central composite designs is presented. For SCCD, the optimal combinations resulting in A, D and G-efficient designs generally put emphasis on replicating the center portion of the SCCD. For RCCD, the optimal combinations resulting in A- and D- efficient designs generally put emphasis on replicating the factorial and center portions of the RCCD. However, with fixed factorial, axial and center portions, G-optimal and efficient designs are attained except for 2^{6-2} design where emphasis is placed on replicating the factorial, axial and the center portions. For OCCD, the optimal combinations resulting in A- optimal and efficient designs generally put emphasis on replicating center portion of the OCCD. The optimal combinations resulting in D- optimal and efficient designs generally put emphasis on replicating the factorial and center portions of the OCCD. To achieve designs that are G-optimal and G-efficient, the optimal combination of design points generally put emphasis on replicating the factorial and the axial portions of the OCCD. For FCCD, the optimal combinations of design points resulting in Aoptimal and A-efficient designs put emphasis on replicating the axial portion of the FCCD. The optimal combinations resulting in D- optimal and efficient designs as well as G-optimal and efficient designs generally put emphasis on replicating the factorial and axial portions of the FCCD. It is interesting to note that for RCCD in 2^{4-1} and 2^{5-1} design variables, any r^{th} complete replicate of the distinct design points of the combination $[r_f{:}\ r_\alpha{:}\ r_c]$ resulted in a G-efficient design. A super-efficient design having efficiency values greater than 100% emerged under the Dcriterion with A and G-efficiency values of 61.91% and 96.49%, respectively indicating that the design performed very well under such combinations as none of the values fell below 50%.

Keywords: SCCD, RCCD, OCCD, FCCD, optimal values, Design efficiency

1 Introduction Many experiments involve the study of the effects of two or more factors. Factorial designs are most efficient for this type of experiment. In a factorial design, all possible combinations of the levels of the factors

ISSN: 3065-0577
International Journal of Data science and Statistics
Research Article

are investigated in each replication. If there are ‘a’ levels of factor A, and ‘b’ levels of factor B, then each replicate contains all ‘ab’ treatment combinations. A factorial design can be either full or fractional factorial. In a Full factorial design (FFD), the effect of all the factors and their interactions on the outcome (s) is investigated. A common experimental design is one, where all input factors are set at two levels each. These levels are termed high and low or +1 and -1, respectively. A design with all possible high/low groupings of all the input factors is termed as a full factorial design in two levels. If there are k factors, each at 2 levels, a full factorial design will be of 2^k runs. To keep experimental costs in line, one approach is to use fractional factorial designs. In these, one does not take measurements upon every possible combination of factor levels, but only upon a very carefully chosen few. These few are selected to ensure that the main effects and low-order interactions can be estimated and tested, at the expense of high-order interactions. Thus, one might design the collection in a Fractional Factorial so that all main effects and two-way interactions can be tested but not three-way or higher-order interactions. The required number of experiments given k factors, is given by

$$N = 2^{K-m} + 2k + n_C \qquad (1.1)$$

for a non-replicated design And

$$N = 2^{K-m} r_f + 2k r_\alpha + n_{Cr} \qquad (1.2)$$

for replicated design

where r_f the number of replications of the factorial point is r_α is the number of replications of the axial point and n_{rc} is the number of replications of the center points

The factorial design is considered a first order model design which consists of constant, linear and interaction terms, and takes the form of

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k-1} \sum_{i<j=2}^{k} \beta_{ij} x_i x_j + \varepsilon \qquad (1.3)$$

Where y is measured response; x_i = the coded independent variables; i =1, 2, k; β’s are unknown parameters and ε is the random error with mean zero and variance σ^2

When there are no replications in a two-level factorial design which involves only continuous factor, the error sum of squares cannot be estimated because there is no degree of freedom available; hence, the model coefficients cannot be statistically tested. In this situation, replication becomes unavoidable. It is necessary to consider a second-order model for the purpose of obtaining a more precise estimate of experimental error and for modeling curvature. Potentially, a SecondOrder polynomial model contains all the terms of the First-Order model, all quadratic terms and all cross-product terms. It is expressed in the form.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j + \sum_{k=1}^{k} \beta_{ii} {x_i}^2 + \varepsilon \qquad (1.4)$$

Where y is measured response; x_i = the coded independent variables; i =1, 2,…, k; β's are unknown parameters and ε is the random error with mean zero and variance σ^2

In this research work, our focus is on one of the most flexible and widely used second-order design - the Central Composite Design. According to Box and Wilson (1951), Central Composite Design is the most popular class of second order designs. They are comprised of replications of 2^k factorial points or 2^{k-p} fractional factorial points of resolution V with replications of the cube, the axial and the center portions. The various replications are denoted in this paper by r_f, r_α and n_{rc} respectively. The CCD is constructed by first creating a 2^k factorial or the 2^{k-p}fraction designs and then appending a set of extra runs referred to as axial or star point. The axial points supply the extra levels required to fit a second order model in all factors. The 2^kor 2^{k-p} portion of the design allows us to fit first order terms and interactions, the axial portion allows us to fit quadratic terms in the factors. So, typically, one first runs factorial portion with center point, this lets us estimate main effects and two-way interactions, and the center point lets us test for curvature. Replication of the center runs allows us to generate a pure error. In order to obtain a better estimate of all linear and product term coefficient, squared term coefficient and also to estimate pure error, it is necessary to repeat the cube, the axial and the center portions a few or several times, this is referred to as replication in Central Composite Design. Replication in the Central Composite Design may take the form of Complete or Partial replications. In this work, equal and partial replications of the cube, the star and the center points are employed. Equal replication implies replicating the cube point, the axial point and the center point equally while partial replication involves replicating either the cube point or the axial point or the center point.

Our focus is to locate the optimum combination of replications of the different portions, i.e., the cube, the axial and the center portions that yield optimal designs. To achieve this, a total of twentyseven (27) equal and partial replicated random experiments for k = 4, 5 and 6 for the various portions of the varying CCD's, namely, Spherical Central Composite Design (SCCD), Rotatable Central Composite Design (RCCD), Orthogonal Central Composite Design (OCCD) and Face Centered Central Composite Design (FCCD). An appropriate experimental design is based on finding the best optimality criterion in which larger efficiency value implies a better design; Boonorm and Borkowski (2012). The adoption of an appropriate experimental design capable of representing the response surface design greatly influences the efficiency of the experimental design Francis and Lilian (2018). Many researchers have carried out different works on replication of Central Composite Designs. Chigbu and Ukaegbu (2017) examined earlier and later studies on the partial replication of the response surface central composite designs (CCDs). The results showed that the optimum performance of the replicated variations of the CCD depends on the axial distance, α, and also the cuboidal or spherical design region when the factorial and axial points are replicated in both design regions studied, no particular replicated variation of the CCD is consistently optimum. They concluded that in most cases replicating the axial points, improves the designs. Ibanga, (2013) compared some variations of experimental points of central composite designs in the presence of complete replication under rotatable and orthogonal design restrictions using the A-, D- and E- optimality criteria The efficiency values obtained suggest that replicated cubes plus replicated star points are better than partial replication of cube and star points under the design restrictions of rotatability and orthogonally.

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

Iwundu (2015) carefully examined the optimal partially replicated cube, star and center runs in Face centered central composite designs. The cube points were replicated while holding the star points and center points were not replicated, the star points was replicated while the cube points and star points were not replicated and the center points were replicated while holding the cube points and the star points fixed. The efficiencies of the designs were assessed using the D and G optimality criteria. The results showed that with the Face centered central composite design replicating the cube portion twice with fixed star and center points performed better than other variations under D and G-optimality criteria. It was also observed that replicating the cube points were more efficient than replicating the center points, and as such, emphasis should shift from replication of only center points, as non-center points performed better. Two variations of central composite designs under the orthogonal and rotatable restriction using the D optimality criterion were compared by Chigbu and Ohaegbulem (2011) and they concluded that the replicated cube plus one star variation is better than the replicated star plus one cube variation under both restrictions. Nduka and Chigbu (2014) compared two variation of N point orthogonal and rotatable central Composite design based on Schur's ordering of design which says

$$\sum_{i=1}^{k} \lambda_i(\xi) \geq \sum_{i=1}^{k} \lambda_i(\eta);\ (\mathrm{k} = 1,2,\ldots,\mathrm{p}) \qquad (1.4)$$

as well as the D optimality and A optimality criteria. They came to the conclusion that by Schur's ordering of designs; a fixed axial point with replicated cube point is better than a fixed cube point plus replicated axial point. It was further demonstrated that the result remained the same for both A-optimality and D-Optimality.

Francis and Lilian (2018) investigated the effect of replicating the cube point, the axial point or center point and the results suggest that replication affects the different criteria in different dimensions because what improves one criterion positively may have negative impact on different criterion. It was further suggested that, experimenters should be willing to sacrifice design efficiency to gain pure error degree of freedom for lack of fit in the case of a decrease in efficiency of the replicated star or cube portion.

Iwundu (2017) studied the effects of addition of n_C center points on the optimality of Box-Benhken and Box-Wilson second-order designs. Relationships were seen to exist between optimal design properties and varying size of the designs by the addition of center points, the relationships

Between the Box-Benhken designs and the central composite design defined at α = $\sqrt{k}$ and α = $f^{\frac{1}{4}}$ are very strong and variations seem to exist with central composite designs defined at α = 1

2. Methodology

sIn this work, equal replication and partial replication for the various central composite designs; Spherical, Rotatable, Orthogonal, and Face centered central composite designs were employed to determine at what level of combination of the replication of the factorial point, the axial point and the center point, an optimum can be attained using the A, D and G optimality criteria. This was computed using some statistical Software packages, namely, DESIGN EXPERT trial version 13 and MATLAB R2021. This study covers the Central Composite Design with k = 4, 5 and 6 design variables.

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

2.1 Model and Design

The Central composite design (CCD) emanated from the response surface designs and is the most popular and commonly used classes of experimental design for fitting a second-order response surface model and is given as

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k-1} \sum_{=i+1}^{k} \beta_{ij} x_i x_j + \sum_{k=1}^{k} \beta_{ii} {x_i}^2 + \varepsilon \qquad (2.1)$$

Where y is measured response; x_i = the coded independent variables; $i^{\cdot}$=1, 2, k; β's are unknown parameters and ε is the random error with mean zero and variance σ^2

The Central Composite Design is made up of the cube portion, the axial or star portion and the center runs. The four varieties of the Central Composite Design discussed in this research paper are: Spherical Central Composite Design (SCCD), the Rotatable Central Composite Design (RCCD), the Orthogonal Central Composite Design (OCCD) and the Face Centered Central Composite Design (FCCD).

The Spherical Central Composite design is a design that puts all the factorial and axial design point on the surface of a sphere of radius $\sqrt{k}$, and one of the major features is rotatability.

Rotatable Central Composite Design is a design that has the same prediction variance for any defined point equidistant from the design center. The alpha value is calculated as $\alpha = F^{\frac{1}{4}}$

Where F is the number of factorial or fractional factorial points.

Orthogonal Central Composite Design plays an important role as a second-order design. A design is said to be orthogonal if the effect of any factor balances out (totals up to zero) across the effects of other factors or if the off-diagonal elements of the information matrix $X'X$ are zero. As in Khuri (1996), the condition for making a CCD to be orthogonal is by setting

$$\alpha = \left[\frac{\sqrt{Nf} - f}{2}\right]^{\frac{1}{2}}$$

The Face Centered Cube Design shall also be explored. This type of design places the axial point at the center of each face of the factorial space. Setting α = 1 makes the CCD Face Centered Cube Design. For given second-order model, an N x p model matrix shall be formed using the design and the model. For example, the model matrix associated with central composite design with no replications and ink design variables for axial distance α and design size, N is represented in Algebraic form as

X =

$$\left| 1 \quad x_{11} \quad x_{12} \quad \cdots \quad x_{1k} \quad x_{11}^2 \quad x_{12}^2 \quad \cdots \quad x_{1k}^2 \quad x_{11}x_{12} \quad x_{11}x_{13} \quad \cdots \quad x_{1(k-1)}x_{1k} \right|$$

ISSN: 3065-0577
International Journal of Data science and Statistics
Research Article

1	x_{21}	x_{22}	···	x_{2k}	x_{21}^2	x_{22}^2	···	x_{2k}^2	$x_{21}x_{22}$	$x_{21}x_{23}$	···	$x_{2(k-1)}x_{2k}$
1	x_{31}	x_{32}	···	x_{3k}	x_{31}^2	x_{32}^2	···	x_{3k}^2	$x_{31}x_{32}$	$x_{31}x_{33}$	···	$x_{3(k-1)}x_{3k}$
1	x_{n1}	x_{n2}	···	x_{nk}	x_{n1}^2	x_{n2}^2	···	x_{nk}^2	$x_{n1}x_{n2}$	$x_{n1}x_{n3}$	···	$x_{n(k-1)}x_{nk}$
⋮	⋮	⋮	⋮	⋮	⋮			⋮	⋮	⋮	⋮	
1	-α	0	···	0	α^2	0	···	0	0	0	···	0
1	α	0	···	0	α^2	0	···	0	0	0	···	0
1	0	-α	···	0	0	α^2	···	0	0	0	···	0
1	0	α	···	0	0	α^2	···	0	0	0	···	0
1	0	0	···	-α	0	0	···	α^2	0	0	···	0
	⋮	⋮	α	⋮	⋮	⋮	α^2	⋮	⋮	⋮	⋮	
1	0	0	···	0	0	0	···	0	0	0	···	0

The Information Matrix denoted by J in this research work is given as

$J = X^TX$

The inverse of the information matrix denoted by Z in this research work is written as

$Z = (X^TX)^{-1}$

The Normalized Information Matrix otherwise known as the Moment Matrix denoted by M is given as

$M = \frac{X^TX}{N}$

Where N the total number of runs is used as a penalty for the larger design.

2.2 Optimality Criteria

In studying the design efficiencies of replicated central composite designs with full factorial portion, the A, D and G design efficiencies shall be employed to assess the quality of the designs. According to Chernoff (1953), the A-Optimality criterion seeks to minimize the trace of the inverse of the information matrix ($X'X$). *A*-efficiency is directly related to minimizing the individual variances of the model parameters; it provides a way of comparing designs across different sample sizes.

2.2.1 A-Optimality

This criterion introduced by Chemoff (1953) seeks to minimize the trace of the inverse of the information matrix X^TX, that is A_{opt} = {min [trace $(X^TX)^{-1}$]} (2.1) Where X is the design matrix and trace is the sum of the design elements of the matrix. The Aefficiency is given as

$$A_{eff} = \frac{100p}{trace[N(X'X)^{-1}]} \tag{2.2}$$

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

2.2. D-Optimality

The D-optimality criterion was the first alphabetical optimality criterion established, according to Wald (1943), this is based on the determinant of XTX which is inversely proportional to the square of the volume of the confidence region on the regression coefficients. It indicates how well the set of coefficients are estimated. Therefore, a smaller $|X^TX|$ or equivalently, a larger $|(X^TX)^{-1}|$ implies poorer estimation of the regression coefficient in the model. The goal of D-optimality is to maximize $|X^TX|$ or equivalently minimize $|(X^TX)^{-1}|$where X is the design matrix. Mathematically, D_{opt} = [max| (X^TX)|] or [min| $(X^TX)^{-1}$|] (2.3)

The D-efficiency according to Crosier (1993) is the pth root of the ratio of $\frac{\det(X'X)}{N^P}$ to maximum possible value of $\frac{\det(X'X)}{N^P}$ for any design defined on the same region. The D-efficiency is

$$D_{eff} = 100\frac{|X'X|^{\frac{1}{P}}}{N} \qquad (2.4)$$

2.2.3. G-Optimality

The aim of G-optimality criterion is to minimize the maximum prediction variance in the design region. Hence,

$$G_{opt} = min[N\hat{\sigma}^2_{max}] \qquad (2.5)$$

The G-efficiency of a design is defined as ^{p}

$$G = \frac{}{V(x)_{max}} \qquad (2.6)$$

Where p is the number of parameters in the model and V(x) max is maximum scaled variance of prediction. The variance of the function at$x$ according to Myer (1966) is

$$V(\hat{y}(x)) = \frac{V(x)\sigma^2}{N} \qquad (2.7)$$

Where V(x) = N$\underline{x}$ $(X'X)^{-1}\underline{x}$ is the scaled prediction variance for any point $\underline{x}$ in the design region.

Thus

Var $(\hat{y}(x))$ =$\underline{\underline{x}}'M^{-1}\underline{\underline{x}}$.

The vector x is the row vector of the design matrix X, associated with the design point $\underline{x}$. =

G-efficiency thus examines the maximum value of V $(\underline{x}) = \frac{\text{NVar}(\hat{y}(x))}{\sigma^2}$ within the design region with respect to its theoretical minimum variance p. Therefore, a G-optimality and the corresponding G-efficiency emphasize the use of designs for which the maximum $\frac{Nvar[\hat{y}(x)]}{\sigma^2}$ in the region of the design is not too large.

3 Results

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

In this research work, the optimal A, D and G-optimality values were obtained for equal and partial replication of cube, axial and center points using selected varieties of the Central Composite Designs; the SCCD, RCCD, OCCD and FCCD, for factor k = 4, 5 and 6. Various combinations of replications of the cube, the axial and the center portions were employed to track the exact points where optimal values occur as can be seen in the tables be

Table 1: Optimality properties for SCCD with Fractional Factorial Replicates in k = 4-1 variables

Design type: SCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$ max$\left|\left(\frac{X'X}{N}\right)\right|$	**min**$\left|\left(\frac{X'X}{N}\right)^{-}\right|$	$Gopt$	$Aeff$	$Deff$	$Geff$
1	17	12	2.000	1	1	1	33.73	0.1208	8.2796	17.00	35.57	83.85	70.59
2	18	12	2.000	1	1	2	23.47	0.1217	8.2198	12.38	49.04	83.90	96.97
3	19	12	2.000	1	1	3	21.87	0.0954	10.4844	13.06	54.87	82.22	91.87
4	25	12	2.000	1	2	1	46.22	0.0717	13.9429	25.00	25.96	80.29	48.00
5	26	12	2.000	1	2	2	31.82	0.0896	11.1615	15.17	37.71	81.71	79.12
6	27	12	2.000	1	2	3	27.42	0.0854	11.7036	15.75	43.76	81.47	66.44
7	33	12	2.000	1	3	1	59.04	0.0365	27.4330	33.00	20.33	75.88	36.36
8	34	12	2.000	1	3	2	39.58	0.0510	19.6256	18.06	30.32	78.03	66.44
9	35	12	2.000	1	3	3	40.74	0.0360	27.7903	18.59	35.87	78.41	64.54
10	25	12	2.000	2	1	1	42.71	0.0717	13.9429	25.00	28.10	80.29	48.00

11	26	12	2.000	2	1	2	28.17	0.0896	11.1615	15.17	42.60	81.79	79.12
12	27	12	2.000	2	1	3	23.64	0.0854	11.7036	15.75	50.80	81.47	76.19
13	33	12	2.000	2	2	1	53.37	0.0864	11.5733	33.00	22.49	81.54	36.36
14	34	12	2.000	2	2	2	33.73	0.1208	8.2796	17.00	35.57	83.85	70.59
15	35	12	2.000	2	2	3	27.44	0.1279	7.8160	12.03	43.74	84.25	99.74
16	41	12	2.000	2	3	1	54.58	0.0658	15.2010	41.00	18.58	79.71	29.27
17	42	12	2.000	2	3	2	39.90	0.0985	10.1491	21.00	30.08	82.44	57.14
18	43	12	2.000	2	3	3	31.89	0.1114	8.9736	14.33	37.63	83.29	83.72
19	33	12	2.000	3	1	1	52.85	0.0365	27.4330	33.00	22.71	75.88	36.36
20	34	12	2.000	3	1	2	33.20	0.0510	19.6256	18.06	36.14	78.03	66.44
21	35	12	2.000	3	1	3	26.89	0.0540	18.5269	18.59	44.63	78.41	64.54
22	41	12	2.000	3	2	1	62.65	0.0658	15.2010	41.00	19.15	79.71	29.27
23	42	12	2.000	3	2	2	37.93	0.0985	10.1491	21.00	31.64	82.44	57.14
24	43	12	2.000	3	2	3	29.88	0.1114	8.9736	14.33	40.17	78.03	83.72
25	49	12	2.000	3	3	1	73.25	0.0651	73.25	49.00	16.38	79.64	24.49
26	50	12	2.000	3	3	2	43.49	0.1021	9.7926	25.00	27.59	82.69	48.00

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

27	51	12	2.000	3	3	3	33.73	0.1208	8.2796	17.00	35.57	83.85	70.59

Table 2: Optimality properties for SCCD with Fractional Factorial Replicates in k = 5-1 variables

Design type: SCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	D opt max$\left\|\left(\frac{X'X}{N}\right)\right\|$	min$\left\|\left(\frac{X'X}{N}\right)^{-1}\right\|$	$Gopt$	$Aeff$	$Deff$	$Geff$
1	27	21	2.236	1	1	1	56.84	0.0093	107.8450	27.00	36.95	80.02	77.78
2	28	21	2.236	1	1	2	42.14	0.0086	115.7309	23.96	49.83	79.75	87.64
3	29	21	2.236	1	1	3	37.85	0.0062	161.2121	24.82	55.49	78.50	84.62
4	37	21	2.236	1	2	1	74.35	0.0014	715.0307	37.00	28.25	73.13	56.76
5	38	21	2.236	1	2	2	53.56	0.0016	625.9131	30.08	39.21	73.59	69.81
6	39	21	2.236	1	2	3	47.17	0.0014	719.9859	30.88	44.52	73.10	60.02
7	47	21	2.236	1	3	1	92.34	2.0274 X 10^{-4}	4.9324 X 10^{3}	47.00	22.74	66.70	44.68
8	48	21	2.236	1	3	2	65.51	2.6059 X 10^{-4}	3.8374 X 10^{3}	24.00	32.06	67.50	87.50
9	49	21	2.236	1	3	3	57.07	2.5351 X 10^{-4}	3.9446 X 10^{3}	37.02	36.80	67.41	56.73
10	43	21	2.236	2	1	1	73.80	0.0096	103.9976	43.00	28.46	80.16	48.84
11	44	21	2.236	2	1	2	49.12	0.0119	84.2679	23.89	42.75	80.96	87.92
12	45	21	2.236	2	1	3	41.23	0.0111	90.0591	24.43	50.93	80.71	85.97
13	53	21	2.236	2	2	1	87.58	0.0069	145.6647	53.00	23.98	78.89	39.62
14	54	21	2.236	2	2	2	56.84	0.0093	107.8450	27.00	36.95	80.02	77.78
15	55	21	2.236	2	2	3	46.89	0.0095	105.6954	23.53	44.79	80.10	89.23
16	63	21	2.236	2	3	1	102.2 5	0.0026	377.5987	63.00	20.54	75.38	33.33
17	64	21	2.236	2	3	2	65.48	0.0038	262.8018	32.00	32.07	76.70	65.63

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

18	65	21	2.236	2	3	3	53.50	0.0041	242.6265	26.60	39.25	76.99	78.94
19	59	21	2.236	3	1	1	93.10	0.0050	199.5651	59.00	22.56	77.71	39.59
20	60	21	2.236	3	1	2	58.68	0.0070	142.0165	30.21	35.79	78.98	69.52
21	61	21	2.236	3	1	3	47.46	0.0075	133.9670	30.71	44.25	79.20	68.38
22	69	21	2.236	3	2	1	105.2 1	0.0078	128.6571	69.00	19.96	79.35	30.44
23	70	21	2.236	3	2	2	64.74	0.0115	87.0226	35.00	32.44	80.84	60.00
24	71	21	2.236	3	2	3	51.46	0.0128	78.1461	23.67	40.81	81.60	88.73
25	79	21	2.236	3	3	1	118.6 3	0.0052	191.3836	79.00	17.71	77.86	26.58
26	80	21	2.236	3	3	2	72.13	0.0080	124.6222	40.00	29.11	79.47	52.50
27	81	21	2.236	3	3	3	56.84	0.0093	107.8450	27.00	36.95	80.02	77.78

Table 3: Optimality properties for SCCD with Fractional Factorial Replicates in k = 6-1 variables

Design type: SCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	D opt		$Gopt$	$Aeff$	$Deff$	$Geff$
								max $\left\lvert\left(\frac{X'X}{N}\right)\right\rvert$	**min** $\left\lvert\left(\frac{X'X}{N}\right)^{-}\right\rvert$				
1	45	28	2.450	1	1	1	83.03	0.0072	139.1304	45.0 0	33.73	83.8 4	62.2 2
2	46	28	2.450	1	1	2	58.04	0.0078	128.7245	28.8 8	48.25	84.0 7	96.9 5
3	47	28	2.450	1	1	3	50.16	0.0064	156.7078	29.5 1	55.82	83.4 8	94.8 9
4	57	28	2.450	1	2	1	101.47	0.0017	601.8678	57.0 0	27.59	79.5 7	49.1 2
5	58	28	2.450	1	2	2	69.42	0.0020	489.7326	34.4 4	40.33	80.1 6	81.3 1

6	59	28	2.45 0	1	2	3	59.15	0.0019	526.9146	35.0 3	47.34	79.9 4	79.9 3
7	69	28	2.45 0	1	3	1	120.7 0	2.3336 X 10^{-4}	4.2852 X 10^{3}	69.0 0	23.20	74.1 8	40.5 8
8	70	28	2.45 0	1	3	2	81.61	3.1195 X 10^{-4}	3.2056 X 10^{3}	40.0 2	34.31	74.9 5	69.9 7
9	71	28	2.45 0	1	3	3	68.97	3.1455 X 10^{-4}	3.1791 X 10^{3}	40.5 9	40.60	74.9 8	68.9 9
10	77	28	2.45 0	2	1	1	119.4 8	0.0032	315.0361	77.0 0	23.44	81.4 3	36.3 6
11	78	28	2.45 0	2	1	2	75.53	0.0044	226.0679	39.6 9	37.07	82.4 0	70.5 5
12	79	28	2.45 0	2	1	3	61.13	0.0046	215.3066	40.2 0	45.80	82.5 4	69.6 6
13	89	28	2.45 0	2	2	1	134.0 2	0.0049	203.5083	89.0 0	20.89	82.7 1	31.4 6
14	90	28	2.45 0	2	2	2	83.03	0.0072	139.1304	45.0 0	33.73	83.8 4	62.2 2
15	91	28	2.45 0	2	2	3	66.25	0.0079	126.3862	30.3 3	42.26	84.1 3	92.3 1
16	101	28	2.45 0	2	3	1	73.53	4.2703 X 10^{-4}	2.3417 X 10^{3}	101. 0	38.08	75.8 0	27.7 2
17	102	28	2.45 0	2	3	2	74.26	3.2408 X 10^{-4}	3.0856 X 10^{3}	51.0 0	37.71	75.0 6	54.9 0
18	103	28	2.45 0	2	3	3	72.93	0.0046	218.1778	34.3 3	38.39	82.5 0	81.5 5

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

19	109	28	2.45 0	3	1	1	157.9 9	9.6611 X 10-4	1.0351 X 10^3	109. 0	17.72	78.0 4	25.6 9
20	110	28	2.45 0	3	1	2	95.27	0.0015	668.3378	55.0 0	29.39	79.2 7	50.9 1
21	111	28	2.45 0	3	1	3	74.56	0.0017	574.0543	53.4 5	37.56	79.7 0	52.3 9
22	121	28	2.45 0	3	2	1	170.4 9	0.0035	288.0529	121. 0	16.42	81.6 9	23.1 4
23	122	28	2.45 0	3	2	2	100.7 3	0.0055	181.3540	61.0 0	27.80	83.0 5	45.9 0
24	123	28	2.45 0	3	2	3	77.64	0.0066	151.9511	41.0 0	36.06	83.5 8	68.2 9
25	133	28	2.45 0	3	3	1	185.2 4	0.0036	274.8172	133. 0	15.12	81.8 3	21.0 5
26	134	28	2.45 0	3	3	2	108.4 7	0.0059	169.4737	67.0 0	25.82	83.2 5	41.7 9
27	135	28	2.45 0	3	3	3	83.03	0.0072	139.1304	45.0 0	33.73	83.8 4	62.2 2

Table 4: Optimality properties for SCCD with Fractional Factorial Replicates in k = 6-2 variables

Design type: SCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left|\left(\frac{X'X}{N}\right)\right|$	**min**$\left|\left(\frac{X'X}{N}\right)^{-}\right|$				
1	29	20	2.450	1	1	1	54.92	0.2378	4.2047	29.0 0	36.4 2	93.07	68.9 7
2	30	20	2.450	1	1	2	39.31	0.2415	4.1416	20.6 2	50.8 7	93.14	96.9 8
3	31	20	2.450	1	1	3	34.60	0.1880	5.3197	21.3 1	57.8 1	91.98	93.8 5

4	41	20	2.450	1	2	1	73.51	0.0908	11.0104	41.0 0	27.2 1	88.70	48.7 8
5	42	20	2.450	1	2	2	50.81	0.1122	8.9142	25.7 2	39.3 7	89.64	77.7 5
6	43	20	2.450	1	2	3	43.66	0.1051	9.5142	26.3 4	45.8 1	89.35	75.9 4
7	53	20	2.450	1	3	1	92.53	0.0255	39.2177	53.0 0	21.6 1	83.24	37.7 4
8	54	20	2.450	1	3	2	62.78	0.0351	28.4977	30.8 9	31.8 6	84.58	64.7 4
9	55	20	2.450	1	3	3	53.25	0.0365	27.4219	31.4 7	37.5 6	84.74	63.5 6
10	45	20	2.450	2	1	1	71.78	0.1099	9.0975	45.0 0	27.8 7	89.55	44.4 4
11	46	20	2.450	2	1	2	46.54	0.1416	7.0599	26.4 9	42.9 8	90.69	75.5 1
12	47	20	2.450	2	1	3	38.41	0.1382	7.2361	27.0 6	52.0 7	90.58	73.9 0
13	57	20	2.450	2	2	1	87.22	0.1684	5.9388	57.0 0	22.9 3	91.48	35.0 9
14	58	20	2.450	2	2	2	54.92	0.2378	4.2047	29.0 0	36.4 2	93.07	68.9 7
15	59	20	2.450	2	2	3	44.40	0.2534	3.9457	20.2 8	45.0 5	93.37	98.6 2
16	69	20	2.450	2	3	1	103.4 5	0.1090	9.1701	69.0 0	19.3 3	89.51	28.9 9
17	70	20	2.450	2	3	2	64.11	0.1636	6.1140	35.0 0	31.2 0	91.34	57.1 4
18	71	20	2.450	2	3	3	51. 22	0.1847	5.4130	23.6 7	39.0 5	91.90	84.5 1
19	61	20	2.450	3	1	1	90.57	0.0375	26.6574	61.0 0	22.0 8	84.86	32.7 9

20	62	20	2.450	3	1	2	55.89	0.0542	18.4512	33.0 7	35.7 9	86.44	60.4 8
21	63	20	2.450	3	1	3	44.54	0.0590	16.9399	33.6 0	44.9 1	86.81	59.5 2
22	73	20	2.450	3	2	1	104.6 0	0.1185	8.4356	73.0 0	19.1 2	89.89	27.4 0
23	74	20	2.450	3	2	2	62.87	0.1806	5.5368	37.0 0	31.8 1	91.80	54.0 5
24	75	20	2.450	3	2	3	49.13	0.2071	4.8279	25.0 0	40.7 1	92.43	80.0 0
25	85	20	2.450	3	3	1	119.7 7	0.1262	7.9223	85.0 0	16.7 0	90.17	23.5 3
26	86	20	2.450	3	3	2	71.01	0.1998	5.0051	43.0 0	28.1 7	92.26	46.5 1
27	87	20	2.450	3	3	3	54.92	0.2378	4.2047	29.0 0	36.4 2	93.07	68.9 7

Table 5: Optimality properties for RCCD with Fractional Factorial Replicates in k = 41variables

Design type: RCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max| $\left(\frac{X'X}{N}\right)$ |	**min**| $\left(\frac{X'X}{N}\right)^{-}$				
1	17	12	1.682	1	1	1	36.57	0.0021	484.2620	12.9 9	39.5 6	67.38	92.3 6
2	18	12	1.682	1	1	2	31.21	0.0018	557.6693	13.6 6	47.1 1	66.47	87.8 3
3	19	12	1.682	1	1	3	29.71	0.0013	751.3729	14.3 8	50.1 6	64.79	83.4 6
4	25	12	1.414	1	2	1	30.68	4.050 X 10^{-4}	2.469 X 10^{3}	18.4 7	39.1 1	52.16	64.9 8
5	26	12	1.414	1	2	2	29.72	3.219 X 10^{-4}	3.106 X 10^{3}	19.0 4	40.3 8	51.18	63.0 4

6	27	12	1.414	1	2	3	29.38	2.485 X 10^{-4}	4.024 X 10^{3}	19.6 6	40.8 4	50.08	61.0 6
7	33	12	1.189	1	3	1	35.60	1.426 X 10^{-5}	7.012 X 10^{4}	26.0 0	33.7 1	39.47	49.2 4
8	34	12	1.189	1	3	2	35.99	1.117 X 10^{-5}	8.954 X $1O^{4}$	25.0 0	33.3 5	38.67	48.0 0
9	35	12	1.189	1	3	3	36.48	8.736 X 10^{-6}	1.145 X 10^{5}	25.6 5	32.9 0	37.89	46.7 9
10	25	12	2.000	2	1	1	42.71	0.0717	13.9429	25.0 0	28.1 0	80.29	48.0 0
11	26	12	2.000	2	1	2	28.17	0.0896	11.1615	15.1 7	42.6 0	81.79	79.1 2
12	27	12	2.000	2	1	3	23.64	0.0854	11.7036	15.7 5	50.8 0	81.47	76.1 9
13	33	12	1.682	2	2	1	37.28	0.0082	121.2753	17.2 3	32.2 0	67.04	69.6 3
14	34	12	1.682	2	2	2	34.34	0.0088	113.9700	12.9 9	39.5 6	67.38	92.3 6
15	35	12	1.682	2	2	3	27.17	0.0083	120.1494	13.3 2	44.1 7	67.09	90.1 1
16	41	12	1.495	2	3	1	31.45	0.0014	695.8073	15.6 3	38.1 5	57.98	76.7 8
17	42	12	1.495	2	3	2	29.52	0.0013	749.8391	15.8 9	40.6 5	57.62	75.5 3
18	43	12	1.495	2	3	3	28.35	0.0012	833.6263	16.1 8	42.3 2	57.11	74.1 6
19	33	12	2.213	3	1	1	42.07	0.2179	4.5890	26.0 0	28.5 4	75.88	46.1 6
20	34	12	2.213	3	1	2	28.93	0.2724	3.6715	19.4 3	41.5 0	89.75	61.7 6
21	35	12	2.213	3	1	3	24.02	0.2771	3.6083	19.8 9	49.9 0	89.77	60.3 4

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

22	41	12	1.861	3	2	1	56.21	0.0238	41.9403	34.4 1	21.3 4	73.25	34.8 7
23	42	12	1.861	3	2	2	37.06	0.0328	30.4551	19.1 7	32.3 7	75.24	62.6 1
24	43	12	1.861	3	2	3	30.10	0.0361	27.7377	13.4 7	39.8 6	75.82	89.0 7
25	49	12	1.682	3	3	1	41.67	0.0077	130.0090	20.6 5	28.8 1	66.65	58.1 0
26	50	12	1.682	3	3	2	34.34	0.0086	116.5248	14.8 3	35.0 6	67.26	80.9 4
27	51	12	1.682	3	3	3	30.34	0.0088	113.9700	12.9 9	39.5 6	67.38	92.3 6

Table 6: Optimality properties for RCCD with Fractional Factorial Replicates in k = 5-1 variables

Design type: RCCD	**N**	**P**	**α**	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	**min**$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	27	21	2.00 0	1	1	1	51.94	0.0012	865.8060	23.8 1	40.43	72.46	88.2 0
2	28	21	2.00 0	1	1	2	42.15	9.5670 X 10^{-4}	1.0453 X 10^{3}	24.6 5	49.83	71.82	85.2 1
3	29	21	2.00 0	1	1	3	38.90	6.5818 X 10^{-4}	1.5193 X 10^{3}	25.5 1	53.98	70.55	82.3 3
4	37	21	1.68 2	1	2	1	47.85	8.4948 X 10^{-6}	1.1772 X 10^{5}	32.0 3	43.89	57.34	65.5 6
5	38	21	1.68 2	1	2	2	46.30	6.1662 X 10^{-6}	1.6217 X 10^{5}	32.7 9	45.36	56.48	64.0 4
6	39	21	1.68 2	1	2	3	45.62	4.3352 X 10^{-6}	2.3067 X 10^{5}	33.5 9	46.03	55.54	62.5 3

7	47	21	1.52 0	1	3	1	52.38	1.7040 X 10-7	5.8686 X 106	40.0 7	40.09	47.60	52.4 1
8	48	21	1.52 0	1	3	2	52.39	1.2472 X 10-7	8.0178 X 106	40.8 3	40.08	46.90	51.4 3
9	49	21	1.52 0	1	3	3	52.61	9.0756 X 10-8	$1.1019 X10^7$	41.6 1	39.92	46.19	50.4 7
10	43	21	2.37 8	2	1	1	66.75	0.0359	27.8309	38.2 8	31.48	85.37	54.8 6
11	44	21	2.37 8	2	1	2	46.38	0.0419	23.8532	24.7 9	45.29	86.00	84.7 3
12	45	21	2.37 8	2	1	3	39.38	0.0385	25.9927	25.2 9	53.34	85.65	83.0 4
13	53	21	2.00 0	2	2	1	67.10	0.0010	956.8079	33.7 3	31.30	72.12	62.2 6
14	54	21	2.00 0	2	2	2	51.94	0.0012	865.8060	23.8 1	40.43	72.46	88.2 0
15	55	21	2.00 0	2	2	3	45.53	0.0011	916.4328	24.2 2	46.12	72.27	86.7 0
16	63	21	1.80 7	2	3	1	52.28	9.0411 X 10-5	1.1061 X 104	27.6 0	40.16	64.19	76.0 8
17	64	21	1.80 7	2	3	2	48.17	8.3097 X 10-5	1.2034 X 104	27.9 5	43.59	63.93	75.1 3
18	65	21	1.80 7	2	3	3	45.71	7.3106 X 10-5	1.3679 X 104	28.3 3	45.94	63.55	74.1 3
19	59	21	2.63 2	3	1	1	53.17	0.2242	4.4609	35.0 2	39.50	93.14	59.9 7
20	60	21	2.63 2	3	1	2	48.90	0.2339	4.2752	34.9 0	48.91	93.32	60.1 9
21	61	21	2.63 2	3	1	3	37.92	0.2193	4.5599	35.1 0	55.38	93.04	59.8 3
22	69	21	2.21 3	3	2	1	108.9 4	1.5514 X 10-4	6.4460e+03	68.6 0	19.99	78.60	30.6 1

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

23	70	21	2.21 3	3	2	2	64.85	0.0094	106.8355	34.9 0	32.38	80.07	60.1 7
24	71	21	2.21 3	3	2	3	51.62	0.0104	96.0341	23.6 2	40.69	80.48	88.9 0
25	79	21	2.00 0	3	3	1	77.78	9.4010 X 10^{-4}	1.0637 X 103	42.5 4	27.00	71.76	49.3 7
26	80	21	2.00 0	3	3	2	60.22	0.0011	900.4481	28.0 0	34.87	72.33	75.0 0
27	81	21	2.00 0	3	3	3	51.94	0.0012	865.8060	23.8 1	40.43	72.46	88.2 0

Table 7: Optimality properties for RCCD with Fractional Factorial Replicates in k = 6-1 variables

Design type: RCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	**min**$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	45	28	2.37 8	1	1	1	82.24	0.0036	276.2210	43.7 1	34.0 5	81.81	64.0 6
2	46	28	2.37 8	1	1	2	58.28	0.0039	259.2697	29.0 2	48.0 5	82.00	96.4 9
3	47	28	2.37 8	1	1	3	50.58	0.0032	317.1662	29.6 5	55.3 6	81.41	94.4 5
4	57	28	2.00 0	1	2	1	1.079	2.6089 X 10^{-5}	3.8330 X 10^{4}	35.4 8	45.5 1	68.60	78.9 3
5	58	28	2.00 0	1	2	2	56.75	2.1375 X 10^{-5}	4.6783 X 10^{4}	36.0 2	49.3 4	68.11	77.7 3
6	59	28	2.00 0	1	2	3	54.16	1.6556 X 10^{-5}	6.0402 X 10^{4}	36.6 0	51.7 0	67.49	76.5 1
7	69	28	1.80 7	1	3	1	59.01	4.1999 X 10^{-7}	2.3810 X 10^{6}	42.4 4	47.4 4	59.20	65.9 7

8	70	28	1.80 7	1	3	2	58.14	3.2319 X 10-7	3.0942 X 106	43.0 0	48.1 6	58.65	65.1 2
9	71	28	1.80 7	1	3	3	57.62	2.4580 X 10-7	4.0683 X 106	43.5 6	48.5 9	58.08	64.2 8
10	77	28	2.82 8	2	1	1	71.17	0.1713	5.8368	43.7 9	39.3 7	93.91	63.9 4
11	78	28	2.82 8	2	1	2	57.16	0.1791	5.5825	43.5 5	49.0 0	94.06	64.2 9
12	79	28	2.82 8	2	1	3	50.34	0.1672	5.9803	43.7 0	55.6 4	93.83	64.0 9
13	89	28	2.37 8	2	2	1	129.1 0	0.0025	394.1553	84.0 5	21.6 8	80.79	33.3 1
14	90	28	2.37 8	2	2	2	82.24	0.0036	276.2210	43.7 1	34.0 5	81.81	64.0 6
15	91	28	2.37 8	2	2	3	65.49	0.0054	186.6401	29.7 5	42.2 8	82.06	94.1 2
16	101	28	2.14 9	2	3	1	83.95	2.3817 X 10-4	4.1987 X 103	41.5 5	33.3 5	74.24	67.4 0
17	102	28	2.14 9	2	3	2	70.21	2.5507 X 10-4	3.9205 X 103	31.9 4	39.8 8	74.42	87.6 6
18	103	28	2.14 9	2	3	3	62.82	2.5065 X 10-4	3.9896 X 103	32.2 2	44.5 7	74.37	86.9 1
19	109	28	3.13 0	3	1	1	52.56	1.0673	0.9370	62.2 6	53.2 9	100.2 4	44.9 7
20	110	28	3.13 0	3	1	2	48.20	1.0060	0.9941	61.8 7	58.1 0	100.0 3	45.2 6
21	111	28	3.13 0	3	1	3	45.23	0.9201	1.0868	61.7 5	61.9 1	99.71	45.3 4
22	121	28	2.63 2	3	2	1	125.8 7	0.0267	37.3925	84.5 6	22.2 6	87.88	33.1 2
23	122	28	2.63 2	3	2	2	86.34	0.0361	27.7082	50.1 9	32.4 4	88.82	55.7 9

24	123	28	2.63 2	3	2	3	69.99	0.0405	24.6714	35.8 5	40.0 1	89.19	78.1 1
25	133	28	2.37 8	3	3	1	173.6 3	0.0019	517.7150	122. 2	16.1 2	80.01	22.9 1
26	134	28	2.37 8	3	3	2	105.9 5	0.0030	332.9157	64.1 7	26.4 2	81.28	43.6 4
27	135	28	2.37 8	3	3	3	82.24	0.0036	276.2210	43.7 1	34.0 5	81.81	64.0 6

Table 8: Optimality properties for RCCD with Fractional Factorial Replicates in k = 6-2 variables

Design type: RCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$ max$\left\|\left(\frac{X'X}{N}\right)\right\|$	$Dopt$ min$\left\|\left(\frac{X'X}{N}\right)^{-}\right.$	$Gopt$	$Aeff$	$Deff$	$Geff$
1	29	20	2.000	1	1	1	42.25	0.0025	401.6639	21.5 2	47.3 4	74.10	92.9 2
2	30	20	2.000	1	1	2	37.66	0.0019	527.5172	22.1 9	53.1 1	73.10	90.1 4
3	31	20	2.000	1	1	3	35.78	0.0013	762.2663	22.8 9	55.8 9	71.76	87.3 9
4	41	20	1.682	1	2	1	42.33	1.6065×10^{-5}	6.2249×10^{4}	29.4 0	47.2 5	57.58	68.0 3
5	42	20	1.682	1	2	2	42.10	1.1567×10^{-5}	8.6450×10^{4}	30.0 4	47.5 1	56.64	66.5 8
6	43	20	1.682	1	2	3	42.13	8.2535×10^{-6}	1.2116×10^{5}	30.7 0	47.4 7	55.69	65.1 8
7	53	20	1.495	1	3	1	49.22	2.0424×10^{-7}	4.8963×10^{6}	37.2 4	40.6 4	46.31	53.7 0
8	54	20	1.495	1	3	2	49.68	1.5244×10^{-7}	6.5600×10^{6}	37.8 9	40.2 7	45.63	52.7 9

9	55	20	1.495	1	3	3	49.68	1.5244 X 10^{-7}	6.5600 X 10^{6}	38.5 4	39.8 6	44.97	51.8 9
10	45	20	2.378	2	1	1	70.98	0.0552	18.1296	43.7 2	28.1 7	86.53	45.7 5
11	46	20	2.378	2	1	2	46.77	0.0701	14.2755	26.1 7	42.7 7	87.57	76.4 1
12	47	20	2.378	2	1	3	38.83	0.0680	14.7038	26.7 3	51.5 1	87.44	74.8 2
13	57	20	2.000	2	2	1	47.28	0.0026	378.2150	21.2 3	42.3 0	74.32	94.2 2
14	58	20	2.000	2	2	2	42.25	0.0025	401.6639	21.5 2	47.3 3	74.10	92.9 2
15	59	20	2.000	2	2	3	39.41	0.0022	452.3068	21.8 4	50.7 5	73.66	91.5 4
16	69	20	1.778	2	3	1	41.28	1.4038 X 10^{-4}	7.1236 X 10^{3}	25.3 0	48.4 4	64.19	79.0 5
17	70	20	1.778	2	3	2	40.38	1.2002 X 10^{-4}	8.3322 X 10^{3}	25.6 1	49.5 3	63.69	78.0 9
18	71	20	1.778	2	3	3	39.78	1.0147 X 10^{-4}	9.8548 X 10^{3}	25.9 4	50.2 9	63.15	77.1 2
19	61	20	2.632	3	1	1	76.17	0.2455	4.0729	50.1 9	26.2 6	93.23	39.8 5
20	62	20	2.632	3	1	2	50.80	0.3233	3.0928	34.4 5	39.3 8	94.52	58.0 6
21	63	20	2.632	3	1	3	41.39	0.3408	2.9344	34.8 8	48.3 3	94.77	57.3 4
22	73	20	2.213	3	2	1	74.96	0.0152	65.7689	45.4 6	26.6 6	81.13	44.0 0
23	74	20	2.213	3	2	2	55.06	0.0188	53.2324	28.4 0	36.3 1	81.99	70.4 3
24	75	20	2.213	3	2	3	46.35	0.0199	50.3278	22.2 4	43.1 4	82.22	89.9 5

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

25	85	20	2.000	3	3	1	49.86	0.0026	378.3968	21.2 5	40.1 1	74.32	94.1 2
26	86	20	2.000	3	3	2	45.24	0.0026	382.4970	21.3 2	44.2 1	74.28	94.8 1
27	87	20	2.000	3	3	3	42.25	0.0025	401.6639	21.5 2	47.3 3	74.10	92.9 2

Table 9: Optimality properties for OCCD with Fractional Factorial Replicates in k = 4-1 variables

Design type: OCCD	N	P	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								$\max\left\|\left(\frac{X'X}{N}\right)\right\|$	$\min\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	17	12	1.353	1	1	1	28.12	5.583×10^{-4}	1.791×10^{3}	14.2 1	42.6 7	53.57	84.4 7
2	18	12	1.414	1	1	2	26.75	6.753×10^{-4}	1.481×10^{3}	14.7 5	44.8 6	54.43	81.3 6
3	19	12	1.471	1	1	3	25.82	7.811×10^{-4}	1.280×10^{3}	16.8 2	46.4 8	55.10	89.1 7
4	25	12	1.239	1	2	1	31.45	6.784×10^{-5}	1.474×10^{4}	19.4 5	38.1 7	44.95	61.7 1
5	26	12	1.267	1	2	2	31.15	6.814×10^{-5}	1.468×10^{4}	19.9 6	38.5 2	44.96	60.1 2
6	27	12	1.294	1	2	3	30.96	6.797×10^{-5}	1.471×10^{4}	20.4 7	38.7 6	44.94	58.6 1
7	33	12	1.173	1	3	1	35.95	1.175×10^{-5}	8.514×10^{4}	24.5 0	33.6 7	38.81	48.9 8
8	34	12	1.190	1	3	2	35.97	1.131×10^{-5}	8.844×10^{4}	25.0 0	33.3 6	38.69	48.0 1
9	35	12	1.207	1	3	3	36.01	1.093×10^{-5}	9.151×10^{4}	25.4 9	33.3 1	38.58	47.0 8
10	25	12	1.414	2	1	1	31.19	0.0011	889.6820	14.0 0	38.4 8	56.79	85.7 2

11	26	12	1.483	2	1	2	28.36	0.0017	591.9348	14.1 5	42.3 1	58.73	84.8 3
12	27	12	1.547	2	1	3	26.28	0.0024	422.6494	14.5 1	45.6 5	60.41	82.7 0
13	33	12	1.321	2	2	1	29.03	4.971 X 10^{-4}	2.012 X 10^{3}	13.9 3	41.3 3	53.04	86.1 3
14	34	12	1.353	2	2	2	28.12	5.578 X 10^{-4}	1.793 X 10^{3}	14.2 1	42.6 7	53.57	84.4 7
15	35	12	1.384	2	2	3	27.37	6.175 X 10^{-4}	1.620 X 10^{3}	14.4 8	43.8 4	54.03	82.8 8
16	41	12	1.266	2	3	1	29.89	1.806 X 10^{-4}	5.536 X 10^{3}	16.5 9	40.1 4	48.75	72.3 4
17	42	12	1.286	2	3	2	29.49	1.870 X 10^{-4}	5.348 X 10^{3}	16.8 6	40.6 9	48.90	71.1 9
18	43	12	1.306	2	3	3	29.15	1.936 X 10^{-4}	5.166 X 10^{3}	17.1 2	41.1 6	49.03	70.0 9
19	33	12	1.439	3	1	1	36.39	9.286 X 10^{-4}	1.077 X 10^{3}	17.5 3	32.9 8	55.90	68.4 5
20	34	12	1.511	3	1	2	32.27	0.0015	645.6817	17.4 2	37.1 9	58.32	68.9 1
21	35	12	1.578	3	1	3	29.22	0.0024	418.0997	17.6 1	41.0 7	60.49	68.1 5
22	41	12	1.357	3	2	1	30.51	8.240 X 10^{-4}	1.214 X 10^{3}	12.2 2	39.3 4	55.35	98.2 3
23	42	12	1.392	3	2	2	29.13	9.956 X 10^{-4}	1.004 X 10^{3}	12.2 6	41.2 0	56.21	96.8 5
24	43	12	1.425	3	2	3	27.97	0.0012	852.4112	12.4 0	42.9 1	57.00	96.8 1
25	49	12	1.310	3	3	1	29.38	4.763 X 10^{-4}	2.099 X 10^{3}	13.8 4	40.8 5	52.86	86.7 0
26	50	12	1.332	3	3	2	31.38	3.921 X 10^{-4}	2.550 X 10^{3}	14.0 2	41.8 0	53.23	85.5 7

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

27	51	12	1.353	3	3	3	28.12	5.578 X 10^{-4}	1.793 X 10^{3}	14.2 1	42.6 7	53.57	84.4 7

Table 10: Optimality properties for OCCD with Fractional Factorial Replicates in k = 5-1 variables

Design type: OCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	min$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	27	21	1.54 7	1	1	1	43.1 5	1.9766 X 10^{-5}	5.0592 X 10^{4}	25.0 0	48.66	59.7 0	84.0 2
2	28	21	1.60 7	1	1	2	41.6 0	2.2483 X 10^{-5}	4.4477 X 10^{4}	25.7 4	50.48	60.0 8	81.5 9
3	29	21	1.66 4	1	1	3	40.5 2	2.4438 X 10^{-5}	4.0920 X 10^{4}	26.4 8	51.83	60.3 2	79.3 0
4	37	21	1.44 3	1	2	1	47.0 0	6.3643 X 10^{-7}	1.5713 X 10^{6}	32.9 8	44.68	50.7 0	63.6 8
5	38	21	1.47 1	1	2	2	46.8 7	5.8612 X 10^{-7}	1.7061 X 10^{6}	33.7 1	44.81	50.5 0	62.3 0
6	39	21	1.49 8	1	2	3	46.8 3	5.3531 X 10^{-7}	1.8681 X 10^{6}	34.4 3	44.85	50.2 9	60.9 9
7	47	21	1.38 0	1	3	1	53.4 3	3.0060 X 10^{-8}	3.3267 X 10^{7}	40.8 2	39.30	43.8 3	51.4 5
8	48	21	1.39 7	1	3	2	53.6 6	2.6580 X 10^{-8}	3.7622 X 10^{7}	41.5 4	39.14	43.5 8	50.5 6
9	49	21	1.41 4	1	3	3	53.9 2	2.3600 X 10^{-8}	4.2373 X 10^{7}	42.2 5	39.95	43.3 4	49.7 0
10	43	21	1.59 6	2	1	1	49.1 3	6.2301 X 10^{-5}	1.6051 X 10^{4}	23.2 2	42.74	63.0 6	90.4 4

11	44	21	1.66 2	2	1	2	45.5 2	9.3650 X 10-5	1.0678 X 10^4	23.1 4	46.13	64.2 9	90.7 5
12	45	21	1.72 4	2	1	3	42.7 7	1.3167 X 10-4	7.5946 X 10^3	23.3 1	49.11	65.3 6	90.0 8
13	53	21	1.51 5	2	2	1	44.1 6	1.8125 X 10-5	5.5173 X 10^4	24.6 2	47.56	59.4 6	85.2 9
14	54	21	1.54 7	2	2	2	43.1 5	1.9766 X 10-5	5.0592 X 10^4	25.0 0	48.66	59.7 0	84.0 2
15	55	21	1.57 7	2	2	3	42.3 1	2.1152 X 10-5	4.7278 X 10^4	25.3 7	49.64	59.9 1	82.7 8
16	63	21	1.46 6	2	3	1	44.8 3	3.4647 X 10-6	2.8863 X 105	28.6 4	46.84	54.9 6	73.3 2
17	64	21	1.48 6	2	3	2	44.4 7	3.4489 X 10-6	2.8995e+05	29.0 1	47.23	54.9 5	72.3 9
18	65	21	1.50 6	2	3	3	44.1 6	3.4323 X 10-6	2.9135 X 105	29.3 8	47.56	54.9 3	71.4 7
19	59	21	1.61 5	3	1	1	58.1 6	4.9457 X 10-5	2.0220 X 10^4	27.7 9	36.11	62.3 7	75.5 6
20	60	21	1.68 3	3	1	2	52.7 4	8.4107 X 10-5	1.1890 X 10^4	30.1 8	39.82	63.9 7	69.5 9
21	61	21	1.74 8	3	1	3	48.5 3	1.3434 X 10-4	7.4435 X 10^3	30.0 9	43.27	65.4 1	69.7 9
22	69	21	1.54 5	3	2	1	47.0 3	4.2146 X 10-5	2.3727 X 10^4	21.7 9	44.66	61.9 0	96.3 9
23	70	21	1.57 8	3	2	2	45.3 9	5.0037 X 10-5	1.9985 X 10^4	22.0 4	46.27	62.4 1	95.2 9
24	71	21	1.61 1	3	2	3	43.9 7	5.8939 X 10-5	1.6967 X 10^4	22.2 9	47.76	62.8 9	94.2 2

25	79	21	1.50 4	3	3	1	44.5 4	1.7510 X 10^{-5}	5.7110 X 10^4	24.5 0	47.15	59.3 7	85.7 2
26	80	21	1.52 6	3	3	2	43.8 0	1.8750 X 10^{-5}	5.3334 X 10^4	24.7 5	47.94	59.5 5	84.8 6
27	81	21	1.54 7	3	3	3	43.1 5	1.9766 X 10^{-5}	5.0592 X 10^4	25.0 0	48.66	59.7 0	84.0 2

Table 11: Optimality properties for OCCD with Fractional Factorial Replicates in k = 6-1 variables

Design type: OCCD	**N**	**P**	**α**	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	**min**$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	45	28	1.72 4	1	1	1	55.33	7.9640 X 10^{-6}	1.2557 X 10^5	29.5 8	50.6 0	65.75	94.6 5
2	46	28	1.78 4	1	1	2	52.85	1.0626 X 10^{-5}	9.4110 X 10^4	30.1 3	52.9 8	66.44	92.9 3
3	47	28	1.84 1	1	1	3	50.92	1.3495 X 10^{-5}	7.4103 X 10^4	30.6 8	54.9 9	67.01	91.2 7
4	57	28	1.63 6	1	2	1	54.36	4.9549 X 10^{-7}	2.0182 X 10^6	36.4 8	51.5 1	59.55	76.7 6
5	58	28	1.66 4	1	2	2	53.86	4.9073 X 10^{-7}	2.0378 X 10^6	37.0 1	51.9 9	59.53	75.6 4
6	59	28	1.69 2	1	2	3	53.46	4.8551 X 10^{-7}	2.0597 X 10^6	37.5 5	52.3 8	59.50	74.5 6
7	69	28	1.58 1	1	3	1	58.33	2.5309 X 10^{-8}	3.9511 X 10^7	43.2 8	47.5 0	53.53	64.7 0
8	70	28	1.59 8	1	3	2	58.32	2.3192 X 10^{-8}	4.3118 X 10^7	43.8 1	48.0 2	53.39	63.9 1
9	71	28	1.61 6	1	3	3	58.33	2.1572 X 10^{-8}	4.6355 X 10^7	44.3 4	48.0 0	53.23	63.1 4

10	77	28	1.76 1	2	1	1	69.65	8.6963 X 10^{-6}	1.1499 X 10^5	40.2 4	40.2 0	65.95	69.5 9
11	78	28	1.82 4	2	1	2	64.38	1.4851 X 10^{-5}	6.7333 X 10^4	39.7 8	43.4 9	67.23	70.3 9
12	79	28	1.88 5	2	1	3	60.17	2.4083 X 10^{-5}	4.1523 X 10^4	39.6 4	46.5 4	68.40	70.6 3
13	89	28	1.69 4	2	2	1	56.84	6.7841 X 10^{-6}	1.4740 X 10^5	29.3 1	49.2 6	65.36	95.5 3
14	90	28	1.72 4	2	2	2	55.33	7.9640 X 10^{-6}	1.2557 X 10^5	29.5 8	50.6 0	65.75	94.6 5
15	91	28	1.75 5	2	2	3	54.01	9.3080 X 10^{-6}	1.0743 X 10^5	29.8 6	51.8 4	66.11	93.7 8
16	101	28	1.65 3	2	3	1	54.30	2.0666 X 10^{-6}	4.8389 X 10^5	32.7 7	51.5 7	62.66	85.4 4
17	102	28	1.67 3	2	3	2	53.66	2.1766 X 10^{-6}	4.5943 X 10^5	33.0 4	52.1 8	62.78	84.7 4
18	103	28	1.69 3	2	3	3	53.09	2.2882 X 10^{-6}	4.3702 X 10^5	33.3 2	52.7 4	62.88	84.0 5
19	109	28	1.77 4	3	1	1	86.53	3.6611 X 10^{-6}	2.7314 X 10^5	55.4 3	32.3 6	63.95	50.5 2
20	110	28	1.83 9	3	1	2	78.67	6.9699 X 10^{-6}	1.4347 X 10^5	54.4 2	35.5 9	65.43	51.4 5
21	111	28	1.90 1	3	1	3	72.34	1.2443 X 10^{-5}	8.0366 X 10^4	53.8 7	38.7 0	66.81	51.9 7
22	121	28	1.71 6	3	2	1	64.19	8.6038 X 10^{-6}	1.1623 X 10^5	32.8 6	43.6 2	65.93	85.2 0
23	122	28	1.74 8	3	2	2	61.85	1.1005 X 10^{-5}	9.0866 X 10^5	32.6 6	45.2 7	66.51	85.7 3
24	123	28	1.77 9	3	2	3	59.78	1.3811 X 10^{-5}	7.2407 X 10^5	32.5 4	46.8 4	67.07	86.0 5

25	133	28	1.68 3	3	3	1	57.38	6.3594 X 10^{-6}	1.5725 X 10^5	29.2 2	48.7 9	65.22	95.8 3
26	134	28	1.70 4	3	3	2	56.31	7.1517 X 10^{-6}	1.3983 X 10^5	29.4 0	49.7 2	65.50	95.2 3
27	135	28	1.72 4	3	3	3	55.33	7.9640 X 10^{-6}	1.2557 X 10^5	29.5 8	50.6 0	65.75	94.6 5

Table 12: Optimality properties for OCCD with Fractional Factorial Replicates in k = 6-2 variables

Design type: OCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	**max**$\left|\left(\frac{X'X}{N}\right)\right|$	**min**$\left|\left(\frac{X'X}{N}\right)^{-}\right|$				
1	29	20	1.664	1	1	1	39.36	5.7157 X 10^{-5}	1.7496 X 10^4	22.5 1	50.8 1	61.36	88.8 2
2	30	20	1.719	1	1	2	38.15	7.2437 X 10^{-5}	1.3805 X 10^4	23.0 9	52.4 3	62.08	86.6 2
3	31	20	1.771	1	1	3	37.26	8.7640 X 10^{-5}	1.1410 X 10^4	23.6 6	53.6 8	62.67	84.5 2
4	41	20	1.550	1	2	1	43.35	2.6126 X 10^{-6}	3.8276 X 10^5	30.0 4	46.1 4	52.59	66.5 7
5	42	20	1.575	1	2	2	43.23	2.6081 X 10^{-6}	3.8343 X 10^5	30.6 0	46.2 6	52.58	65.3 7
6	43	20	1.599	1	2	3	43.18	2.5733 X 10^{-6}	3.8861 X 10^5	31.1 5	46.3 2	52.55	64.2 1
7	53	20	1.479	1	3	1	49.53	1.5881 X 10^{-7}	6.2967 X 10^6	37.3 6	40.3 7	45.71	53.5 4
8	54	20	1.494	1	3	2	49.70	1.5005 X 10^{-7}	6.6645 X 10^6	37.9 0	40.2 5	45.59	52.7 7
9	55	20	1.509	1	3	3	49.88	1.4218 X 10^{-7}	7.0331 X 10^6	38.4 4	40.1 0	45.47	52.0 3

10	45	20	1.724	2	1	1	44.08	1.2126 X 10^{-4}	8.2469 X 10^3	25.1 2	45.3 7	63.72	79.6 1
11	46	20	1.784	2	1	2	41.35	1.9374 X 10^{-4}	5.1615 X 10^3	25.2 4	48.3 7	65.22	79.2 5
12	47	20	1.841	2	1	3	39.17	2.9225 X 10^{-4}	3.4218 X 10^3	25.5 2	51.0 6	66.59	78.3 6
13	57	20	1.636	2	2	1	40.11	5.0215 X 10^{-5}	1.9914 X 10^4	22.2 3	49.8 7	60.97	89.9 8
14	58	20	1.664	2	2	2	39.36	5.7157 X 10^{-5}	1.7496 X 10^4	22.5 2	50.8 2	61.37	88.8 3
15	59	20	1.692	2	2	3	38.71	6.4836 X 10^{-5}	1.5423 X 10^4	22.8 0	51.6 7	61.74	87.7 1
16	69	20	1.581	2	3	1	41.08	1.1827 X 10^{-5}	8.4553 X 10^4	26.0 3	48.6 8	56.69	76.8 4
17	70	20	1.598	2	3	2	40.82	1.2160 X 10^{-5}	8.2238 X 10^4	26.3 1	49.0 0	56.80	76.0 1
18	71	20	1.616	2	3	3	40.58	1.2670 X 10^{-5}	7.8928 X 10^4	26.5 9	49.2 9	56.90	75.2 1
19	61	20	1.748	3	1	1	51.69	7.4870 X 10^{-5}	1.3357 X 10^4	32.6 6	38.7 0	62.19	61.2 4
20	62	20	1.810	3	1	2	47.60	1.3326 X 10^{-4}	7.5042 X 10^3	32.4 9	42.0 2	64.02	61.5 6
21	63	20	1.870	3	1	3	44.13	2.2539 X 10^{-4}	4.4367 X 10^3	32.5 6	45.1 3	65.70	61.4 3
22	73	20	1.673	3	2	1	42.22	9.4495 X 10^{-5}	1.0583 X 10^4	21.3 2	47.3 7	62.91	93.8 2
23	74	20	1.703	3	2	2	41.01	1.1642 X 10^{-4}	8.5893 X 10^3	21.3 9	48.7 7	63.57	93.5 2
24	75	20	1.732	3	2	3	39.94	1.4093 X 10^{-4}	7.0959 X 10^3	21.5 0	50.0 8	64.19	93.0 2

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

25	85	20	1.627	3	3	1	40.38	4.8251 X 10^{-5}	2.0725 X 10^4	22.1 3	49.5 3	60.82	90.3 8
26	86	20	1.646	3	3	2	39.84	5.2888 X 10^{-5}	1.8908 X 10^4	22.3 2	50.2 0	61.12	89.6 0
27	87	20	1.664	3	3	3	39.36	5.7157 X 10^{-5}	1.7496 X 10^4	22.5 2	50.8 2	61.37	88.8 3

Table 13: Optimality properties for FCCD with Fractional Factorial Replicates in k = 4-1 variables

Design type: FCCD	**N**	**P**	**α**	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	**min**$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	17	12	1.000	1	1	1	43.55	1.251 X 10^{-5}	7.991 X 10^4	15.2 8	27.5 6	39.04	78.5 5
2	18	12	1.000	1	1	2	45.37	7.506 X 10^{-6}	1.332 X 10^5	16.1 6	26.4 5	37.41	74.2 7
3	19	12	1.000	1	1	3	47.32	4.553 X 10^{-6}	2.1970 X 10^5	17.0 4	25.3 6	35.88	70.4 1
4	25	12	1.000	1	2	1	41.17	3.694 X 10^{-6}	2.708 X 10^5	20.8 0	29.1 5	35.26	57.7 1
5	26	12	1.000	1	2	2	42.39	2.563 X 10^{-6}	3.901 X 10^5	21.5 9	28.3 1	34.20	55.5 7
6	27	12	1.000	1	2	3	43.67	1.793 X 10^{-6}	5.578 X 10^5	22.4 0	27.4 8	33.20	53.5 8
7	33	12	1.000	1	3	1	43.29	1.197 X 10^{-6}	8.353 X 10^5	25.8 8	27.7 2	32.10	46.3 8

8	34	12	1.000	1	3	2	44.28	9.044 X 10^{-7}	9.756 X 10^{6}	26.6 1	27.1 0	31.36	45.0 9
9	35	12	1.000	1	3	3	45.31	6.865 X 10^{-7}	1.457 X 10^{6}	27.3 6	26.4 9	30.65	47.0 8
10	25	12	1.000	2	1	1	54.17	2.043 X 10^{-5}	4.895 X 10^{4}	13.3 2	22.1 5	40.66	90.0 6
11	26	12	1.000	2	1	2	55.35	1.514 X 10^{-5}	6.605 X 10^{4}	13.4 5	21.6 8	39.66	89.1 9
12	27	12	1.000	2	1	3	56.73	1.114 X 10^{-5}	8.978 X 10^{4}	13.6 7	21.1 5	38.66	87.8 1
13	33	12	1.000	2	2	1	42.71	1.620 X 10^{-5}	6.175 X 10^{4}	14.8 4	28.1 0	39.88	80.8 8
14	34	12	1.000	2	2	2	43.55	1.251 X 10^{-5}	7.991 X 10^{4}	15.2 8	27.5 6	39.04	78.5 5
15	35	12	1.000	2	2	3	44.43	9.681 X 10^{-6}	1.033 X 10^{5}	15.7 2	27.0 1	38.21	76.3 5
16	41	12	1.000	2	3	1	40.52	8.579 X 10^{-6}	1.166 X 10^{5}	17.6 9	29.6 2	37.83	67.8 4
17	42	12	1.000	2	3	2	41.20	6.906 X 10^{-6}	1.448 X 10^{5}	18.1 1	29.1 3	37.15	66.2 8
18	43	12	1.000	2	3	3	41.91	5.570 X 10^{-6}	1.795 X 10^{5}	18.5 2	28.6 4	36.49	64.7 8
19	33	12	1.000	3	1	1	66.86	1.606 X 10^{-5}	6.227 X 10^{4}	17.0 2	17.9 5	39.86	70.5 0

20	34	12	1.000	3	1	2	67.63	1.339 X 10^{-5}	7.519 X 10^{4}	17.1 0	17.7 4	39.23	70.5 5
21	35	12	1.000	3	1	3	68.67	1.086 X 10^{-5}	9.210 X 10^{4}	17.1 1	17.4 8	38.58	68.1 5
22	41	12	1.000	3	2	1	47.73	2.323 X 10^{-5}	4.304 X 10^{4}	12.6 8	25.1 4	41.10	96.6 1
23	42	12	1.000	3	2	2	48.36	1.920 X 10^{-5}	5.207 X 10^{4}	12.9 9	24.8 2	40.45	92.4 0
24	43	12	1.000	3	2	3	49.06	1.584 X 10^{-5}	6.313 X 10^{4}	13.2 9	24.4 7	39.81	90.2 9
25	49	12	1.000	3	3	1	42.45	1.765 X 10^{-5}	5.666 X 10^{4}	14.6 9	28.2 7	40.17	81.6 8
26	50	12	1.000	3	3	2	42.98	1.486 X 10^{-5}	6.729 X 10^{4}	14.9 8	27.9 2	39.60	80.0 9
27	51	12	1.000	3	3	3	43.55	1.251 X 10^{-5}	7.991 X 10^{4}	15.2 8	27.5 6	39.04	78.5 5

Table 14: Optimality properties for FCCD with Fractional Factorial Replicates in k = 5-1 variables

Design type: FCCD	**N**	**P**	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								max$\left\|\left(\frac{X'X}{N}\right)\right\|$	**min**$\left\|\left(\frac{X'X}{N}\right)^{-}\right\|$				
1	27	21	1.00 0	1	1	1	83.33	1.7252 X 10^{-8}	5.7966 X 10^{7}	26.0 6	25.20	42.69	80.5 9
2	28	21	1.00 0	1	1	2	85.83	9.1477 X 10^{-9}	1.0932 X 10^{8}	27.0 2	24.47	41.42	77.7 3

3	29	21	1.00 0	1	1	3	88.42	4.9090 X 10^{-9}	2.0371 X 10^{8}	27.9 8	23.75	40.21	75.0 6
4	37	21	1.00 0	1	2	1	73.42	1.1665 X 10^{-9}	8.5725 X 10^{8}	34.6 8	28.60	37.55	60.5 6
5	38	21	1.00 0	1	2	2	75.14	7.1682 X 10^{-10}	1.3950 X 10^{9}	35.6 1	27.95	36.69	58.9 7
6	39	21	1.00 0	1	2	3	76.88	4.4472 X 10^{-10}	2.2486 X 10^{9}	36.5 4	27.31	35.87	57.4 7
7	47	21	1.00 0	1	3	1	75.24	9.1604 X 10^{-11}	1.0917 X 10^{10}	42.9 8	27.91	33.27	48.8 6
8	48	21	1.00 0	1	3	2	76.68	6.1993 X 10^{-11}	1.6131 X 10^{10}	43.8 9	27.39	32.65	47.8 5
9	49	21	1.00 0	1	3	3	78.12	4.2230 X 10^{-11}	2.3680 X 10^{10}	44.7 9	26.88	32.06	46.8 8
10	43	21	1.00 0	2	1	1	113.1 9	4.8335 X 10^{-8}	2.0689 X 10^{7}	22.1 9	18.55	44.84	94.6 6
11	44	21	1.00 0	2	1	2	114.9 5	3.3900 X 10^{-8}	2.9499 X 10^{7}	22.2 5	18.27	44.09	94.3 8
12	45	21	1.00 0	2	1	3	116.8 5	2.3688 X 10^{-8}	4.2215 X 10^{7}	22.3 9	17.97	43.34	93.7 8
13	53	21	1.00 0	2	2	1	82.12	2.3782 X 10^{-8}	4.2049 X 10^{7}	25.5 8	25.57	43.35	82.1 1
14	54	21	1.00 0	2	2	2	83.33	1.7252 X 10^{-8}	5.7966 X 10^{7}	26.0 6	25.20	42.69	80.5 9
15	55	21	1.00 0	2	2	3	84.57	1.2545 X 10^{-8}	7.9713 X 10^{7}	26.5 4	24.83	42.05	79.1 4
16	63	21	1.00 0	2	3	1	74.43	6.1256 X 10^{-9}	1.6325 X 10^{8}	29.9 4	28.22	40.64	70.1 4
17	64	21	1.00 0	2	3	2	75.41	4.6263 X 10^{-9}	2.1616 X 10^{8}	30.4 1	27.85	40.10	69.0 5

18	65	21	1.00 0	2	3	3	76.40	3.5036 X 10^{--9}	2.8542 X 10^{8}	30.8 9	27.79	39.57	67.9 9
19	59	21	1.00 0	3	1	1	146.1 8	3.7447 X 10^{--8}	2.6704 X 10^{7}	29.8 8	14.37	44.30	70.2 7
20	60	21	1.00 0	3	1	2	147.4 7	2.9892 X 10^{-8}	3.3454 X 10^{7}	29.7 7	14.24	43.82	70.5 3
21	61	21	1.00 0	3	1	3	148.9 8	2.3656 X 10^{-8}	4.2272 X 10^{7}	29.7 8	14.10	43.34	70.5 2
22	69	21	1.00 0	3	2	1	96.52	5.0752 X 10^{-8}	1.9704 X 10^{7}	22.4 5	21.76	44.94	93.5 6
23	70	21	1.00 0	3	2	2	97.49	4.0277 X 10^{-8}	2.4828 X 10^{7}	22.7 7	21.54	44.45	92.2 3
24	71	21	1.00 0	3	2	3	98.50	3.1951 X 10^{-8}	3.1298 X 10^{7}	23.0 9	21.32	43.96	90.9 4
25	79	21	1.00 0	3	3	1	81.73	2.6480 X 10^{-8}	3.7764 X 10^{7}	25.4 2	25.70	43.57	82.6 2
26	80	21	1.00 0	3	3	2	82.52	2.1363 X 10^{-8}	4.6809 X 10^{7}	25.7 4	25.45	43.13	81.6 0
27	81	21	1.00 0	3	3	3	83.33	1.7252 X 10^{-8}	5.7966 X 10^{7}	26.0 6	25.20	42.69	80.5 9

Table 15: Optimality properties for FCCD with Fractional Factorial Replicates in k = 6-1 variables

Design type: FCCD	N	P	α	r_f	r_α	n_c	$Aopt$	$Dopt$		$Gopt$	$Aeff$	$Deff$	$Geff$
								$\max\left\|\left(\frac{X'X}{N}\right)\right\|$	$\min\left\|\left(\frac{X'X}{N}\right)^{-}\right.$				
1	45	28	1.000	1	1	1	147.5 3	1.7185 X 10^{-10}	5.8191 X 10^{9}	30.4 4	18.9 8	44.80	91.9 9

2	46	28	1.000	1	1	2	150.23	1.0291 X 10^{-10}	9.7169 X 10^{9}	31.1 2	18.6 4	43.99	89.9 9
3	47	28	1.000	1	1	3	150.02	6.1857 X 10^{-11}	1.6166 X 10^{10}	31.7 9	18.3 0	43.19	88.0 8
4	57	28	1.000	1	2	1	111.72	1.9581 X 10^{-11}	5.1070 X 10^{10}	38.0 0	25.0 6	41.45	73.6 9
5	58	28	1.000	1	2	2	113.68	1.2032 X 10^{-11}	8.3110 X 10^{10}	38.6 6	24.6 8	40.82	72.4 2
6	59	28	1.000	1	2	3	115.21	8.3161 X 10^{-12}	1.2025 X 10^{11}	39.3 3	24.3 0	40.21	71.2 0
7	69	28	1.000	1	3	1	104.43	1.4379 X 10^{-12}	6.9546 X 10^{11}	45.3 9	26.8 1	37.76	61.6 9
8	70	28	1.000	1	3	2	104.30	1.4949 X 10^{-12}	6.6896 X 10^{11}	46.0 5	26.4 6	37.28	60.8 1
9	71	28	1.000	1	3	3	107.20	6.9724 X 10^{-13}	1.4342 X 10^{12}	46.7 0	26.1 2	36.80	59.9 5
10	77	28	1.000	2	1	1	227.49	1.7700 X 10^{-10}	5.6496 X 10^{9}	38.9 8	12.3 1	44.85	71.8 3
11	78	28	1.000	2	1	2	229.49	1.3661 X 10^{-10}	7.3203 X 10^{9}	38.9 3	12.2 0	44.43	71.9 3
12	79	28	1.000	2	1	3	231.63	1.0492 X 10^{-10}	9.5314 X 10^{9}	38.9 6	12.0 9	44.02	71.8 8
13	89	28	1.000	2	2	1	146.22	2.2227 X 10^{-10}	4.4991 X 10^{9}	30.1 0	19.1 5	45.21	93.0 2
14	90	28	1.000	2	2	2	147.53	1.7185 X 10^{-10}	5.8191 X 10^{9}	30.4 4	18.9 8	44.80	91.9 9
15	91	28	1.000	2	2	3	147.53	1.3294 X 10^{-10}	5.8191 X 10^{9}	30.7 8	18.8 1	44.39	90.9 8

ISSN: 3065-0577
International Journal of Data science and Statistics
Research Article

16	101	28	1.000	2	3	1	121.46	8.5627 X 10^{-11}	1.1679 X 10^{10}	33.9 1	23.0 5	43.70	82.5 8
17	102	28	1.000	2	3	2	122.48	6.7521 X 10^{-11}	1.4810 X 10^{10}	34.2 4	22.8 6	43.33	81.7 7
18	103	28	1.000	2	3	3	123.52	5.3311 X 10^{-11}	1.8758 X 10^{10}	34.5 8	22.6 7	42.96	80.9 8
19	109	28	1.000	3	1	1	310.13	7.4004 X 10^{-11}	1.3513 X 10^{10}	54.6 4	9.03	43.47	51.2 5
20	110	28	1.000	3	1	2	311.63	6.3465 X 10^{-11}	1.5757 X 10^{10}	54.3 5	8.99	43.23	51.5 2
21	111	28	1.000	3	1	3	313.34	5.4039 X 10^{-11}	1.8505 X 10^{10}	54.1 9	8.94	42.98	51.6 7
22	121	28	1.000	3	2	1	185.75	2.7172 X 10^{-10}	3.6802 X 10^{9}	31.1 8	15.0 7	45.54	89.8 2
23	122	28	1.000	3	2	2	186.84	2.2809 X 10^{-10}	4.3842 X 10^{9}	31.1 8	14.9 9	45.25	89.8 1
24	123	28	1.000	3	2	3	187.97	1.9127 X 10^{-10}	5.2283 X 10^{9}	31.2 0	14.9 0	44.97	89.7 5
25	133	28	1.000	3	3	1	145.79	2.4219 X 10^{-10}	4.1291 X 10^{9}	29.9 9	19.2 1	45.35	93.3 7
26	134	28	1.000	3	3	2	146.65	2.0399 X 10^{-10}	4.9022 X 10^{9}	30.2 2	19.0 9	45.07	92.6 7
27	135	28	1.000	3	3	3	147.53	1.7185 X 10^{-10}	5.8191 X 10^{9}	30.4 4	18.9 8	44.80	91.9 9

Table 16: Optimality properties for FCCD with Fractional Factorial Replicates in k = 6-2 variables

								Dopt				

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

Desig n type: FCC D	N	P	α	r_f	r_α	n_c	$Aopt$	max$\left\lvert\left(\frac{X'X}{N}\right)\right\rvert$	min$\left\lvert\left(\frac{X'X}{N}\right)^{-}\right\rvert$	$Gopt$	$Aeff$	$Deff$	$Geff$
1	29	20	1.000	1	1	1	98.96	1.4823 X 10^{-9}	6.7462 X 10^{8}	24.1 6	20.2 1	36.19	82.7 7
2	30	20	1.000	1	1	2	101.9 8	8.3456 X 10^{-10}	1.1982 X 10^{9}	24.9 9	19.6 1	35.16	80.0 2
3	31	20	1.000	1	1	3	105.0 5	4.7578 X 10^{-10}	2.1018 X 10^{9}	25.8 2	19.0 4	34.19	77.4 5
4	41	20	1.000	1	2	1	84.89	1.6599 X 10^{-10}	6.0243 X 10^{9}	32.7 9	23.5 6	32.43	60.9 9
5	42	20	1.000	1	2	2	86.79	1.0854 X 10^{-10}	9.2128 X 10^{9}	33.5 9	23.0 5	31.75	59.5 5
6	43	20	1.000	1	2	3	88.69	7.1566 X 10^{-11}	1.3973 X 10^{10}	34.3 8	22.5 5	31.10	58.1 7
7	53	20	1.000	1	3	1	85.09	1.9360 X 10^{-11}	5.1653 X 10^{10}	40.9 5	23.5 0	29.13	48.8 4
8	54	20	1.000	1	3	2	86.59	1.3864 X 10^{-11}	7.2128 X 10^{10}	41.7 1	23.1 0	28.65	47.9 5
9	55	20	1.000	1	3	3	88.09	9.9817 X 10^{-12}	1.0018 X 10^{11}	42.4 8	22.7 1	28.18	47.0 8
10	45	20	1.000	2	1	1	136.2 8	2.6281 X 10 9	3.8050 X 10^{8}	23.4 2	14.6 8	37.24	85.3 9
11	46	20	1.000	2	1	2	138.7 3	1.8764 X 10 9	5.3293 X 10^{8}	23.6 1	14.4 2	36.62	84.7 0
12	47	20	1.000	2	1	3	141.2 7	1.3396 X 10 9	7.4650 X 10^{8}	23.8 5	14.1 6	36.00	83.8 6
13	57	20	1.000	2	2	1	97.47	1.9844 X 10 9	5.0392 X 10^{8}	23.7 5	20.5 2	36.72	84.2 2

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

14	58	20	1.000	2	2	2	98.96	1.4823 X 10-9	6.7462 X 10^8	24.1 6	20.2 1	36.19	82.7 7
15	59	20	1.000	2	2	3	100.4 6	1.1105 X 10 9	9.0046 X 10^8	24.5 8	19.9 1	35.67	81.3 8
16	69	20	1.000	2	3	1	87.18	6.7193 X 10-10	1.4883 X 10 9	28.1 4	22.9 4	34.78	71.0 7
17	70	20	1.000	2	3	2	88.31	5.2386 X 10-10	1.9089 X 10 9	28.5 5	22.6 5	34.35	70.0 6
18	71	20	1.000	2	3	3	89.45	4.0950 X 10-10	2.4420 X 10 9	28.9 5	22.3 6	33.93	69.0 8
19	61	20	1.000	3	1	1	176.6 8	1.5517 X 10 9	6.4444 X 10^8	31.1 8	11.3 2	36.27	64.1 5
20	62	20	1.000	3	1	2	178.8 1	1.2418 X 10 9	8.0530 X 10^8	31.2 4	11.1 9	35.87	64.0 2
21	63	20	1.000	3	1	3	181.0 5	9.8945 X 10 9	1.0107 X 10^9	31.3 7	11.0 5	35.46	63.7 5
22	73	20	1.000	3	2	1	115.5 5	3.2767 X 10-9	3.0518 X 10^8	20.5 9	17.3 1	37.65	97.1 4
23	74	20	1.000	3	2	2	116.8 6	2.6392 X 10 9	3.7890 X 10^8	20.8 7	17.1 1	37.25	95.8 3
24	75	20	1.000	3	2	3	118.1 9	2.1273 X 10 9	4.7008 X 10^8	21.1 5	16.9 2	36.85	94.5 6
25	85	20	1.000	3	3	1	96.97	2.1885 X 10-9	4.5694 X 10^8	23.6 1	20.6 2	36.90	84.7 1
26	86	20	1.000	3	3	2	97.96	1.8000 X 10 9	5.5557 X 10^8	23.8 9	20.4 2	36.54	83.7 3
27	87	20	1.000	3	3	3	98.96	1.4823 X 10-9	6.7462 X 10^8	24.1 6	20.2 1	36.19	82.7 7

The best Optimality and the best efficiency values for SCCD, RCCD, OCCD and FCCD full and fractional replicates have been summarized in **Tables 17-19** with best optimal combinations.

Table 17: Optimality Values and Efficiency Values (%) k = 4-1

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

Design Size	Design Variable K	Parameter P	Optimal Combination with corresponding Design Size and Axial Distance for A-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for D-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for G-Optimality Criterion				
SCCD	4-1	12	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			19	2.000	1	1	3	35	2.000	2	2	3	35	2.000	2	2	3
			Aopt value = 2 1.87 (min) Aeff value = 54.87 (max)					Dop t Value : max| M|= 0. 1279 min| M^{-1}|= 7 .8160 Deff value = 84.2 5(max)					Gopt v alue = 12.03(min) Geff v alue = 99.74(max)				
Best Optimal Combination for SCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			18	2.000	1	1	2	18	2.000	1	1	2	18	2.000	1	1	2
			$Aopt$ = 23.47					D_{opt}^{max} = 0.1217; D_{opt}^{min} = 8.2198					$Gopt$ = 12.38				
			$Aeff$ = 49.04					$Deff$ =83.90					$Geff$ = 96.97				
RCCD	4-1	12	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			27	2.000	2	1	3	35	2.213	2	1	3	17	1.682	1	1	1
			Aopt value = 23.6 4(min) Aeff value = 50.80 (max)					Dopt Value: max| M| = 0. 2771 Min| M^{-1}| = 3.6083 Deff v alue = 89.7 7(max)					Gopt value = 12.9 9(min) Geff value = 92.3 6(max)				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C

Best Optimal Combination for RCCD			27	2.000	2	1	3	27	2.000	2	1	3	27	2.000	2	1	3
			$Aopt = 23.64$					$D_{opt}^{max} = 0.0854$ $D_{opt}^{min} = 11.7036$					$Gopt = 15.75$				
			$Aeff = 50.80$					$Deff = 81.47$					$Geff = 76.19$				
OCCD	4-1	12	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			19	1.471	1	1	3	35	1.578	3	1	3	41	1.357	3	2	1
			Aopt value = 25.82 (min) Aeff value = 46.48(max)					Dopt Value: max| M| = 0.0024 Min| M^{-1}| = 418.0997 Deff value = 60.49(max)					Gopt value = 12.22(min) Geff value = 98.23(max)				
Best Optimal Combination for OCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			19	1.471	1	1	3	19	1.471	1	1	3	19	1.471	1	1	3
			$Aopt = 25.82$					$D_{opt}^{max} = 7.811 \text{ X } 10^{-4}$ $D_{opt}^{min} = 1.280 \text{ X } 10^{3}$					$Gopt = 16.82$				
			$Aeff = 46.48$					$Deff = 55.10$					$Geff = 89.17$				
FCCD	4-1	12	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			41	1.000	2	3	1	41	1.000	3	2	1	41	1.000	3	2	1
			Aopt value = 40.52(min) Aeff value = 29.62 (max)					Dopt Value: max| M| = 2.323 X 10^{-5} Min| M^{-1}| = 4.304 X 10^{4} Deff value = 41.10(max)					Gopt value = 12.68(min) Geff value = 96.61(max)				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C

	42	1.000	3	2	1	41	1.000	3	2	1	41	1.000	3	2	1
Best Optimal Combination for FCCD	$Aopt$ = 47.73					D_{opt}^{max}= 2.323 X 10-5 D_{opt}^{min} = 4.304 X 10^4					$Gopt$ = 12.68				
	$Aeff$ = 25.82					$Deff$ = 41.10					$Geff$ = 96.61				

Table 18: Optimality Values and Efficiency Values (%) k = 5-1

Design Size	Design Variable K	Parameter P	Optimal Combination with corresponding Design Size and Axial Distance for A-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for D-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for G-Optimality Criterion				
SCCD	5-1	21	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			29	2.23 6	1	1	3	71	2.23 6	3	2	3	55	2.23 6	2	2	3
			Aopt value = 3 7.85 (min) Aeff value = 55.49 (max)					Dop t Value : max| M|= 0.0128 min| M^{-1}|= 7 8.1461 Deff value = 81.6 0(max)					Gopt v alue = 23.53(min) Geff v alue = 89.23(max)				
Best O ptimal Co mbination fo r SCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			29	2.23 6	1	1	3	29	2.23 6	1	1	3	29	2.23 6	1	1	3
			$Aopt$ = 37.85					D_{opt}^{max} = 0.0062 D_{opt}^{min} = 161.2121					$Gopt$ = 24. 82				
			$Aeff$ = 55.49					$Deff$ = 78.50					$Geff$ = 84.62				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			61	2.63 2	3	1	3	60	2.63 2	3	1	2	27	2.00 0	1	1	1

ISSN: 3065-0577
International Journal of Data science and Statistics
Research Article

RCCD	5-1	21	Aopt value = 37.9 2(min) Aeff value = 55.38 (max)					Dopt Value: max| M| = 0. 2339 Min| M^{-1}| = 4.2752 Deff v alue = 93.3 2(max)					Gopt value = 23.8 1(min) Geff value = 88.2 0(max)				
Best Optimal Combination for RCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			45	2.37 8	2	1	3	45	2.37 8	2	1	3	45	2.37 8	2	1	3
			$Aopt$ = 39.38					D_{opt}^{max} = 0.0385; D_{opt}^{min} = 25.9927					$Gopt$ = 25.29				
			$Aeff$ = 53.34					$Deff$ = 85.65					$Geff$ = 83.03				
OCCD	5-1	21	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			29	1.66 4	1	1	3	61	1.74 8	3	1	3	69	1.54 5	3	2	1
			Aopt value = 40.52 (min) Aeff value = 51.8 3(max)					Dopt Value: max| M| = 1.3434 X 10^{-4} Min| M^{-1}| = 7 4435 X 10^{3} Deff v alue = 65.4 1(max)					Gopt v alue = 21,7 9(min) Geff v alue = 96.3 9(max)				
Best O ptimal Combination fo r O CCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			28	1.60 7	1	1	2	28	1.60 7	1	1	2	28	1.60 7	1	1	2
			$Aopt$ = 41.60					D_{opt}^{max} = 2.2483 X 10^{-5}; D_{opt}^{min} = 4.4477 X 10^{4}					$Gopt$ = 25. 74				
			$Aeff$ = 51.83					$De\ ff$ = 60.08					$Geff$ = 81.59				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			37	1.00 0	1	2	1	69	1.00 0	3	2	1	41	1.00 0	3	2	1

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

FCCD	5-1	21	Aopt value = 73.4 2(min) Aeff value = 28.60 (max)					Dopt Value: max| M| = 5 0752 X 10^{-8} Min| M^{-1}| = 1. 9704 X 10^{7} Deff value = 44.9 4(max)					Gopt v alue = 22.1 9(min) Geff v alue = 94.6 6(max)				
Best Optimal Combination for FCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			27	1.00 0	1	1	1	27	1.00 0	1	1	1	27	1.00 0	1	1	1
			$Aopt = 83.33$					$D_{opt}^{max} = 1.7252 \text{ X } 10^{-8}$ $D_{opt}^{min} = 5.7966 \text{ X } 10^{7}$					$Gopt = 26.05$				
			$Aeff = 25.20$					$Deff = 42.69$					$Geff = 80.59$				

Table 19: Optimality Values and Efficiency Values (%) k = 6-1

Design Size	Design Variab l K	Paramet er P	Optimal Combination with corresponding Design Size and Axial Distance for A-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for D-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for G-Optimality Criterion				
SCCD	6-1	28	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			47	2.45 0	1	1	3	45	2.45 0	1	1	2	46	2.45 0	1	1	2
			Aopt value = 5 0.16 (min) Aeff value = 55.82 (max)					Dop t Value : max| M| = 0.0078 min| M^{-1}| = 128.7245 Deff value = 84.0 7(max)					Gopt v alue = 28.88(min) Geff v alue = 96.95(max)				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C

Best Optimal Combination for SCCD			47	2.45 0	1	1	3	47	2.45 0	1	1	3	47	2.45 0	1	1	3
			$Aopt$ = 50.16					D_{opt}^{max} = 0.0064 D_{opt}^{min} = 156.7078					$Gopt$ = 29. 51				
			$Aeff$ = 55.82					$Deff$ = 83.48					$Geff$ = 94.89				
RCCD	6-1	28	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			111	3.13 0	3	1	3	109	3.13 0	3	1	1	27	2.00 0	1	1	1
			Aopt value = 45.2 3(min) Aeff value = 61.91 (max)					Dopt Value: max| M| = 1.0673 Min| M^{-1}| = 0.9370 Deff value = 100. 24(max)					Gopt value = 29.0 2(min) Geff value = 96.4 9(max)				
Best Optimal Combination for RCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			47	2.37 8	1	1	3	47	2.37 8	1	1	3	47	2.37 8	1	1	3
			$Aopt$ = 50.58					D_{opt}^{max} = 0.0032 D_{opt}^{min} = 317.1662					$Gopt$ = 29. 65				
			$Aeff$ = 55.36					$Deff$ = 81.41					$Geff$ = 94.45				
OCCD	6-1	28	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			47	1.84 1	1	1	3	79	1.88 5	2	1	3	69	1.54 5	3	2	1
			Aopt value = 50.92 (min) Aeff value = 54.9 9(max)					Dopt Value: max| M| = 2.4083 X 10^{-5} Min| M^{-1}| = 4 1523 X 10^{4} Deff value = 68.4 0(max)					Gopt value = 29.22 (min) Geff value = 95.8 3(max)				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			47	1.84 1	1	1	3	47	1.84 1	1	1	3	47	1.84 1	1	1	3

Best Optimal Combination for OCCD			$Aopt$ = 50.92					D_{opt}^{max} = 1.3495 X 10^{-5} D_{opt}^{min} = 7.4103 X 10^{4}					$Gopt$ = 30.68				
			$Aeff$ = 54.99					$Deff$ = 60.08					$Geff$ = 91.27				
FCCD	6-1	28	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			69	1.00 0	1	3	1	121	1.00 0	3	2	1	41	1.00 0	3	2	1
			Aopt value = 104. 43(min) Aeff value = 26.81 (max)					Dopt Value: max| M| = 2.7172 X 10^{-10} Min| M^{-1}| = 3. 6802 X 10^{9} Deff value = 45.5 4(max)					Gopt v alue = 29.2 9(min) Geff v alue = 93.3 7(max)				
Best Optimal Combination for FCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			101	1.00 0	2	3	1	101	1.00 0	2	3	1	101	1.00 0	2	3	1
			$Aopt$ = 121.46					D_{opt}^{max} = 8.5627 X 10^{-11} D_{opt}^{min} = 1.1679 X 10^{10}					$Gopt$ = 33.91				
			$Aeff$ = 23.05					$Deff$ = 43.70					$Geff$ = 82.58				

Table 35 CONTD: Optimality Values and Efficiency Values (%) k = 6-2

Design Size	Design Variabl K	Parameter P	Optimal Combination with corresponding Design Size and Axial Distance for A-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for D-Optimality Criterion					Optimal Combination with corresponding Design Size and Axial Distance for G-Optimality Criterion				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			31	2.45 0	1	1	3	59	2.45 0	2	2	3	59	2.45 0	2	2	3

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

SCCD	6-2	20	Aopt value = 3 4.60 (min) Aeff value = 57.81 (max)					Dop t Value : max| M| = 0. 2534 min| M^{-1}| = 3.9457 Deff value = 93.3 7max)					Gopt v alue = 20.28(min) Geff v alue = 98.62(max)				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			31	2.45 0	1	1	3	31	2.45 0	1	1	3	31	2.45 0	1	1	3
								D_{opt}^{max} = 0.1880									
Best Optimal Combination for SCCD			$Aopt$ 34.60					D_{opt}^{min} = 5.3197					$Gopt$ = 21.31				
			$Aeff$ = 57.81					$Deff$ = 91.98					$Geff$ = 93.85				
RCCD	6-2	20	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			31	2.00 0	1	1	3	63	2.63 2	3	1	3	27	2.00 0	3	3	2
			Aopt value = 35.78 (min) Aeff value = 55.89 (max)					Dopt Value: max| M| = 0.3233 Min| M^{-1}| = 2.9344 Deff v alue = 94.7 7(max)					Gopt value = 21.3 2(min) Geff value = 94.8 1(max)				
Best Optimal Co mbinatio n for RCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			30	2.00 0	1	1	2	30	2.00 0	1	1	2	30	2.00 0	1	1	2
			$Aopt$ = 37. 66					D_{opt}^{max} = 0.0019 D_{opt}^{min} = 527.5172					$Gopt$ = 22. 19				
			$Aeff$ = 53.11					$Deff$ = 73.10					$Geff$ = 90.14				
			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			31	1.77 1	1	1	3	47	1.84 1	2	1	3	73	1.67 3	3	2	1

OCCD	6-2	20	Aopt value = 37.26 (min) Aeff value = 53.68 (max)					Dopt Value: max| M| = 2.9225 X 10^{-4} Min| M^{-1}| = 3 4218 X 10^{3} Deff v alue = 66.5 9(max)					Gopt v alue = 21.3 2(min) Geff v alue = 93.8 2(max)				
Best Optimal Co mbinatio n for OCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			30	1.71 9	1	1	2	30	1.71 9	1	1	2	30	1.71 9	1	1	2
			$Aopt$ = 38. 15					D_{opt}^{max} = 7.2437 X 10^{-5} D_{opt}^{min} = 1.3805 X 10^{4}					$Gopt$ = 23. 09				
			$Aeff$ = 52.43					$Deff$ = 62. 08					$Geff$ = 86.62				
FCCD	6-2	20	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			41	1.00 0	1	2	1	73	1.00 0	3	2	1	73	1.00 0	3	2	1
			Aopt value = 84.89(min)					Dopt Value: max|M| = 3.2767 X 10^{-9} Min|M^{-1}| = 3.0518 X 10^{8}					Gopt value = 20.59(min)				
			Aeff value = 23.56(max)					Deff value = 37.65(max)					Geff value = 97.14(max)				
Best Optimal Combination for FCCD			N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C	N	α	r_f	r_α	n_C
			57	1.00 0	1	1	2	101	1.00 0	2	3	1	101	1.00 0	2	3	1
			$Aopt$ = 97.47					D_{opt}^{max} = 1.9844 X 10^{-9} D_{opt}^{min} = 5.0392 X 10^{8}					$Gopt$ = 23.75				
			$Aeff$ = 20.52					$Deff$ = 36.72					$Geff$ = 84.22				

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

4 Discussion of Results

With reference to Table 1, for SCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 21.87 with A-efficiency value of 54.87%. The associated design is of size N = 19 and contains one complete 24-1 factorial points, one complete 2k axial points and three center points. Optimal combination of [2:2:3] yielded the best Gopt and best D_{opt}^{max} values of 12.03 and 0.1279, respectively with corresponding best G and best D-efficiency values of 99.74% and 84.25%, respectively. The associated design is of size N = 35 and contains two complete 241 factorial points, two complete 2k axial points and three center points. It is interesting to note that the design associated with the [1:1:2] combination resulted in the best optimal combination for A, D and G criteria Similarly, from Table 2, for SCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 37.85 with A-efficiency value of 55.49%. The associated design is of size N = 29 and contains one complete 25-1 factorial points, one complete 2k axial points and three center points. Optimal combination of [2:2:3] yielded the best Gopt value of 23.53 with the best G-efficiency value of 89.23%. The associated design is of size N = 55 and contains two complete 25-1 factorial points, two complete 2k axial points and three center points. Optimal combination of [3:2:3] yielded the best D_{opt}^{max}value of 0.0128 with the best D-efficiency value of 81.60%. The associated design is of size N = 71 and contains three complete 25-1 factorial points, two complete 2k axial points and two center points. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria From Table 3, for SCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 50.16 with A-efficiency value of 55.82%. The associated design is of size N = 47 and contains one complete 26-1 factorial points, one complete 2k axial points and three center points. Optimal combination of [1:1:2] yielded the best Gopt and best D_{opt}^{max} values of 28.88 and 0.0078, respectively with corresponding best G and best D-efficiency values of 96.95% and 84.07%, respectively. The associated design is of size N = 46 and contains one complete 26-1 factorial points, one complete 2k axial points and two center points. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria From Table 4, for SCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 34.60 with A-efficiency value of 57.81%. The associated design is of size N = 31 and contains one complete 26-2 factorial points, one complete 2k axial points and three center points. Optimal combination of [2:2:3] yielded the best Gopt and best D_{opt}^{max} values of 20.28 and 0.2534, respectively with corresponding best G and best D-efficiency values of 96.62% and 93.37%, respectively. The associated design is of size N = 59 and contains two complete 26-2 factorial points, two complete 2k axial points and three center points. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria From Table 5, for RCCD fractional factorial replicates, optimal combination of [2:1:3] yielded the best Aopt value of 23.64 with A-efficiency value of 50.80%. The associated design is of size N = 27 and contains two complete 24-1 factorial points, one complete 2k axial points and three center points. It is interesting to note that optimal combination of [1:1:1], [2:2:2] and [3:3:3] yielded the same best Gopt values of

12.99 and best G-efficiency values of 92.36%. The associated design sizes are N = 17, 34 and 51, respectively. It is also of interest to note that the design associated with the [1:1:1] combination has the smallest design size of N = 17 and as such, exhibits a runsize efficiency advantage over the other combinations, and could be considered to yield the best Gopt and G-efficiency values. Optimal combination of [3:1:3] yielded the best D_{opt}^{max} value of 0.2771 and D-efficiency value 89.77%. The associated design is of size N = 35 and contains three complete 24-1 factorial points, one complete 2k axial points and three center points. The design associated with the [2:1:3] combination resulted in the best optimal combination for A, D and G criteria Similarly, from Table 6, for RCCD fractional factorial replicates, optimal combination of [3:1:3] yielded the best Aopt value of 37.92 with A-efficiency value of 55.38%. The associated design is of size N = 61 and contains three complete 24 factorial points, one complete 2k axial points and three center points. Interestingly, combination of [1:1:1], [2:2:2] and [3:3:3] yielded the same best Gopt values of 23.81 and best G-efficiency values of 88.20%. The associated design sizes are N = 27, 54 and 81, respectively. The design associated with the [1:1:1] combination has the smallest design size of N = 27 and as such, exhibits a run-size efficiency advantage over the other combinations, and could be considered to yield the best Gopt and G-efficiency values. Optimal combination of [3:1:2] yielded the best D_{opt}^{max} value of 0.2339 and D-efficiency value 93.32%. The associated design is of size N = 60 and contains three complete 25-1 factorial points, one complete 2k axial points and two center points. The design associated with the [2:1:3] combination resulted in the best optimal combination for A, D and G criteria From Table 7, for RCCD fractional factorial replicates, optimal combination of [3:1:3] yielded the best Aopt value of 45.23 with A-efficiency value of 61.91%. The associated design is of size N = 111 and contains three complete 26-1 factorial points, one complete 2k axial points and three center points. Optimal combination of [1:1:2] yielded the best Gopt value of 29.02 and best G-efficiency value of 96.49%. The associated design is of size N = 46 and contains one complete 26-1 factorial points, one complete 2k axial points and two center points. Optimal combination of [3:1:1] resulted in the best D_{opt}^{max} value of 1.0673 with super D-efficiency value of 100.24%. The associated design is of size N = 109 and contains three complete 26-1 factorial points, one complete 2k axial points and one center point. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria From Table 8, for RCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 35.78 with A-efficiency value of 55.89%. The associated design is of size N = 31 and contains one complete 26-2 factorial points, one complete 2k axial points and three center points. Optimal combination of [3:3:2] yielded the best Gopt value of 21.32 and best G-efficiency value of 94.81%. The associated design is of size N = 86 and contains three complete 26-2 factorial points, three complete 2k axial points and two center points. Optimal combination of [3:1:3] yielded the best D_{opt}^{max} value of 0.3233 and best D-efficiency value of 94.77%. The associated design is of size N = 63 and contains three complete 48 factorial points, one complete 2k axial points and three center points. The design associated with the [1:1:2] combination resulted in the best optimal combination for A, D and G criteria Similarly, From Table 9, for OCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 25.82 with A-efficiency value of 46.48%. The associated design is of size N = 19 and contains one complete 8 factorial points, one complete 2k axial points and three center points. Optimal combination of [3:2:1] yielded the best Gopt value of 12.22 and best Gefficiency value of 98.23%. The associated design is of size N = 41 and contains three complete 24 factorial points, two complete 2k axial points and one center point. Optimal combination of [3:1:3] yielded the best D_{opt}^{max} value of 2.40 X 10^{-3} and best D-efficiency value of 60.49%. The associated design is of size N = 35 and contains three complete 24 factorial points, one complete 2k axial points and three center points. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria. From Table 10, for OCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 40.53 with A-efficiency value of 51.83%. The associated design is of size N = 29 and contains one complete 16 factorial points, one complete 2k axial points and three center points. Optimal combination of [3:2:1] yielded the best Gopt value of 21.79 and best G-efficiency value of 96.39%. The associated design is of size N = 69 and contains three complete 48 factorial points, two complete 2k axial points and one center point. Optimal combination of [3:1:3] yielded the best D_{opt}^{max} value of 1.34 X 10^{-4} and best D-efficiency value of 65.41%. The associated design is of size N = 61 and contains three complete 48 factorial points, one complete 2k axial points and three center points. The design associated with the [1:1:2] combination resulted in the best optimal combination for A, D and G criteria. From Table 11, for OCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 50.92 with A-efficiency value of 54.99%. The associated design is of size N = 47 and contains one complete 32 factorial points, one complete 2k axial points and three center points. Optimal combination of [3:2:1] yielded the best Gopt value of 29.22 and best G-efficiency value of 95.83%. The associated design is of size N = 69 and contains three complete 48 factorial points, two complete 2k axial points and one center point. Optimal combination of [2:1:3] yielded the best D_{opt}^{max} value of 2.40 X 10^{-5} and best D-efficiency value of 68.40%. The associated design is of size N = 79 and contains two complete 64 factorial points, one complete 2k axial points and three center points. The design associated with the [1:1:3] combination resulted in the best optimal combination for A, D and G criteria. From Table 12, for OCCD fractional factorial replicates, optimal combination of [1:1:3] yielded the best Aopt value of 37.26 with A-efficiency value of 53.68%. The associated design is of size N = 31 and contains one complete 16 factorial points, one complete 2k axial points and three center points. Optimal combination of [3:2:1] yielded the best Gopt value of 21.32 and best G-efficiency value of 93.82%. The associated design is of size N = 73 and contains three complete 48 factorial points, two complete 2k axial points and one center point. Optimal combination of [2:1:3] yielded the best D_{opt}^{max} value of 2.92 X 10^{-4} and best D-efficiency value of 66.59%. The associated design is of size N = 79 and contains two complete 64 factorial points, one complete 2k axial points and three center points. The design associated with the [1:1:2] combination resulted in the best optimal combination for A, D and G criteria. From Table 13, for FCCD fractional factorial replicates, optimal combination of [3:2:1] yielded the best Aopt value of 47.73 with A-efficiency value of 12.68%, with best G-efficiency value of 96.61% and best D_{opt}^{max}value of 2.32 X 10^{-5} with the best D-efficiency value of 41.10%. The associated design is of size N = 41 and contains three complete 24 factorial points, two complete 2k axial points and one center point. It is interesting to note that the design associated with the [3:2:1] combination resulted in the best optimal combination for A, D and G criteria

Similarly, from Table 14, for FCCD fractional factorial replicates, optimal combination of [1:2:1] yielded the best Aopt value of 73.42 with A-efficiency value of 28.60%. The associated design is of size N = 37 and contains one complete 25-1 factorial points, two complete 2k axial points and one center point. Optimal combination of [2:1:1] yielded the best Gopt value of 22.19 and best Gefficiency value of 94.66%. The associated design is of size N = 43 and contains two complete 251 factorial points, one complete 2k axial points and one center point. Optimal combination of [3:2:1] resulted in the best D_{opt}^{max} value of 5.0752 X 10-8 with best D-efficiency value of 44.94%. The associated design is of size N = 69 and contains three complete 25-1 factorial points, two complete 2k axial points and one center point. The design associated with the [1:1:1] combination resulted in the best optimal combination for A, D and G criteria. From Table 15, for FCCD fractional factorial replicates, optimal combination of [1:3:1] yielded the best Aopt value of 104.43 with A-efficiency value of 26.81%. The associated design is of size N = 69 and contains one complete 26-1 factorial points, three complete 2k axial points and one center point. Optimal combination of [3:3:1] yielded the best Gopt value of 19.21 and best Gefficiency value of 93.37%. The associated design is of size N = 133 and contains three complete 26-1 factorial points, three complete 2k axial points and one center point. Optimal combination of [3:2:1] resulted in the best D_{opt}^{max} value of 2.7172X10-10 D-efficiency value of 45.54%. The associated design is of size N = 121 and contains three complete 26-1 factorial points, two complete 2k axial points and one center point. The design associated with the [2:3:1] combination resulted in the best optimal combination for A, D and G criteria From Table 28, for FCCD fractional factorial replicates, optimal combination of [1:2:1] yielded the best Aopt value of 84.89 with A-efficiency value of 23.56%. The associated design is of size N = 41 and contains one complete 26-2 factorial points, two complete 2k axial points and one center point. Note that optimal combination of [3:2:1] yielded the best Gopt and best D_{opt}^{max} values of 20.59 and 3.2767X10-9, respectively with corresponding best G and best D-efficiency values of 97.14% and 37.65%, respectively. The associated design is of size N = 73 and contains three complete 26-2 factorial points, two complete 2k axial points and one center point. The design associated with the [2:3:1] combination resulted in the best optimal combination for A, D and G criteria. It is observed that, A-Optimal design satisfies the criterion A= mintrace [(XTX)-1] and D-Optimal design satisfies the criterion D = min|(XTX)-1| or equivalently D = max|(XTX)|. A design with minimum prediction variance $min[N\hat{\sigma}_{max}^2]$ is G-Optimal. Conversely, an efficient design is a design with high efficiency values. When one has various designs at one's disposal, the most efficient design to choose is the one with larger efficiency values with smaller design size. The overall results for the twenty-seven number combinations of different replications of the various portions of all the CCDs under study show that the efficiency of design is dependent on the optimality values; optimality values tend to influence the efficiency of design either negatively or positively. Larger optimality values depreciate the efficiency of the design while smaller optimality values improve efficiency of the design. The results, however, show that all the points where A, D and G are optimal with maximum efficiency values were all partially replicated. Replicating a complete SCCD, RCCD, OCCD and FCCD with one $r_{f,}\, r_{\alpha}$ and n_c r-times, yielded the same A, D and G-efficiency values. Generally, the best G-efficiencies for the various CCDs recorded overall superior performance of 85% and above efficiency values. The best D-efficiency recorded above 70% for SCCD and RCCD and below 70% for OCCD and FCCD. To obtain optimal values and to have a good and efficient design, there is need to replicate the cube, the axial and the center points. Partial replication of the various portions of the CCD tends to yield better results than equal replications. The best designs in terms of optimal and efficiency values seem to put more emphasis on replication of center point for SCCD, RCCD and OCCD but places more emphasis on replication of the axial point for FCCD.

References

Boonorm, C. and Borkowski, J. J., (2012). "Comparison of response surface designs in a spherical region." *International Journal of Mathematical and Computational Sciences*, vol. 6, pp. 545-548.

Box, G. E. P. and Wilson, K. B. (1951). On the experimental attainment of optimum conditions. *Journal of Royal Statistical Society*, 13(1): 1-45.

Chernoff, H. (1953). Locally optimal designs for estimating parameters, Annals of Mathematical Statistics, 24, 586–602.

Chigbu, P. E. and Ohaegbulem, E. U. (2011). On the preference of replicating factorial runs in restricted second-order designs. *Journal of Applied Sciences*, 11(22): 3732-37.

Chigbu, P. E. and Ukaegbu, E. C. (2017). Recent developments on partial replications of response surface central composite designs: A review. *An International Journal of Statistics Applications and Probability*, 6(1): 91.

Crosier, R. B., (1993). *Response surface design comparisons, Technical Report. U. S. army Edgewood research*. Development and Engineering Center, SCBRD-RTM, Bldg. E3160 Aberdeen proving ground, Maryland, MD 21010-542.

Francis, C. E. & Lilian, N. O. (2018). ‘’Alphabetic optimality criteria for $\mathbf{2^K}$ central composite design’’. *Academic Journal of Applied Mathematical Sciences*, 4, 107-118.

http://arpgweb.com/?ic=journal&journal=17&info=aims

Ibanga, K. D. (*2013). On optimal complete replicated rotatable, Orthogonal, Efficient and relative efficient central composite designs. Research Thesis.*

Iwundu, M. P. (2015). Optimal partially replicated cube, star and center runs in face – centered central composite designs. International Journal of Statistics and Probability, 4(4): 1-19.

ISSN: 3065-0577

International Journal of Data science and Statistics

Research Article

Iwundu, M. P., 2017. "The effects of addition of center points on the optimality of Box-

Benhken and Box-Wilson Second-Order Designs." *International Journal of Probability and Statistics*, vol. 6, pp. 20-32.

Khuri, A. I. and Cornell, J. A., 1996. Response surfaces, Designs and analyses. 2nd ed. Marcel Dekker, Inc.

Myers, R.H., Khuri. A.I. & Center W.H., Jr. (1966-1988). Response surface methodology.

Nduka, U. C. & Chigbu, P. E. (2014). "On optimal choice of cube and star replication in restricted second order designs communication in statistics-theory and methods". *Journal Communications in Statistics Theory and Methods*, 43, 4195-4214.

V. V Fedorov, Theory of Optimal Experiments, Academic Press, New York, 1972 Wald, A., (1943)."On the efficient design of statistical investigations." *Ann. Math. Statist.*, *Statist.*, 14, 134-140.