

DECIPHERING THE FORMULA: SIMPLIFYING THE UNDERSTANDING OF HOLDING PERIOD RETURNS

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Abstract

The concept of holding period return (R) is a fundamental measure in finance, representing the ratio of future proceeds to the initial investment. For bonds, this calculation is defined as $R = (B_1 - B_0 + iF)/B_0$, where B_t denotes the bond valuations at time t , iF represents interest payments at the interest rate i on face value F , and M signifies maturity, discounted at rate k , known as the yield to maturity. Corporate bonds often entail semi-annual interest payments, equivalent to half the annual iF amount. These interest payments can be conceptualized as an annuity, $iF/k(1 - 1/[1+k]^M)$, while the face value is $F/(1+k)^M$. This abstract delves into the mathematical intricacies of holding period returns for bonds and provides insights into their underlying principles.

Keywords: Holding Period Return, Bond Valuation, Yield to Maturity, Corporate Bonds, Interest Payments

Introduction

A holding period return R is a ratio of future proceeds divided by its initial investment. For a bond it is $R = (B_1 - B_0 + iF)/B_0$ with bond valuations B_t at time t with interest payments of iF at interest rate i on face value F , and with a maturity M discounted at rate k which for bonds is called the yield to maturity. Interest payments on corporate bonds are often paid twice a year as half the annual amount of iF . The interest payments are an annuity as $iF/k(1 - 1/[1+k]^M)$ and the face value is $F/(1+k)^M$ or:

$B_0 = iF/k(1 - 1/[1+k]^M) + F/(1+k)^M$ and $B_1 = iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1}$. Thus:

$R = \{iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1} - iF/k(1 - 1/[1+k]^M) - F/(1+k)^M + iF\} / \{iF/k(1 - 1/[1+k]^M) + F/(1+k)^M\}$, canceling F :

$R = \{i/k(1 - 1/[1+k]^{M-1}) + 1/(1+k)^{M-1} - i/k(1 - 1/[1+k]^M) - 1/(1+k)^M + i\} / \{i/k(1 - 1/[1+k]^M) + 1/(1+k)^M\}$, expanding the annuities:

$R = \{i/k - [i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} - i/k + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\}$, canceling like terms:

$R = \{-[i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\}$, multiplying by $(1+k)^M$:

$R = [-i(1+k)/k + (1+k) + i/k - 1 + i(1+k)^M] / [i(1+k)^M/k - i/k + 1]$, and multiplying by k :

$R = [-i(1+k) + (1+k)k + i - k + ik(1+k)^M] / [i(1+k)^M - i + k]$, and expanding:

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$R = [-i - ik + k + kk + i - k + ik(1+k)^M] / [i(1+k)^M - i + k]$, and canceling:

$$R = [-ik + kk + ik(1+k)^M] / [i(1+k)^M - i + k].$$

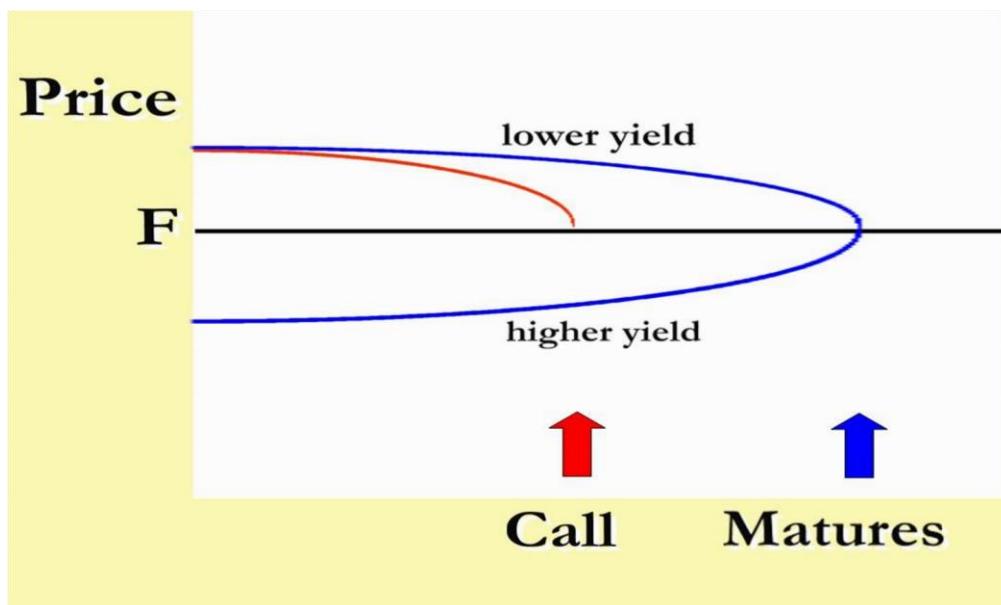
The numerator is a multiple of the denominator by k , therefore $R = k$.

Here are some different examples of holding period returns for various bonds discounted at a 10 percent yield:

Coupon

Interest	Maturity	Price	Maturity	Price	Yield					
0	2	86.64	1	90.91	10					
1	3	77.62	2	84.38	10					
5		30		52.87	29					
10		20		100.00	19					
12		5		107.58	4					
10										

Do be wary of bonds trading at a premium and/or convertible bonds. For bonds trading at a premium, yield to call price and call date would be more appropriate.



For a common stock the holding period return R with valuations P_t and D_t at time t , the yearly return would be $R = (P_1 - P_0 + D_1)/P_0$. Often dividends are paid four times a year with an associated decrease in value on its exdividend date; exchange traded funds often pay monthly. A common stock with a dividend payment of D_t and discounted at rate k with growth rate g , is valued as:

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$$P_0 = D_1/(k-g) = D_0(1+g)/(k-g);$$

Substituting: $R = (D_1[1+g]/[k-g] - D_1/[k-g] + D_1)/(D_1/[k-g])$, Canceling D_1 :

$R = ([1+g]/[k-g] - 1/[k-g] + 1)/(1/[k-g])$, Multiplying by $(k-g)$: $R = (1 + g - 1 + k - g)$ Which results in $R = k$.
Here are some examples of different common stocks with different dividends and different growth rates discounted at a 10 percent discount rate:

Growth Rate	Current Dividend	Current Price	Next Dividend	Next Price	Next Yield
0	2.00	20.00	2.00	20.00	10
4	1.00	17.33	1.04	18.03	10
6	4.00	106.00	4.24	112.36	10
8	1.00	54.00	1.08	58.32	10

Do be wary that tax considerations were neutral here whereas in reality capital gains tax rates and the tax rates upon dividends may differ.

Note that $D_1 = E_1(1-b)$ where E is a firm's earnings, b is the firm's retention rate, and a firm's endogenous growth rate g may be determined by $g = br$ where r is the firm's rate of return. Earnings are achieved on the firm's assets A or $E_1 = A_0r$.

Thus:

$$P_0 = D_1/(k-g) = E_1(1-b)/(k-g) = A_0r(1-b)/(k-br).$$

Likewise $k = D_1/Pg = E_1(1-b)/P+br = A_0r(1-b)/P+br$. Where r is greater (less) than k , P will be valued greater (less) than A . However, when r is greater than k , a lesser dividend and a greater retention rate resulting in increased growth will increase valuations, whereas when r is less than k then a greater dividend and lesser retention and lower growth rates will increase valuations albeit still below asset valuation A . Consider A_0 equaling 100, with r equaling .12 and E_1 equaling 12, the valuations are:

b	1-b	g	D_1	$k=.14$	$k=.12$	$k=.10$
0	1	.00	12	85.7	100.0	120.0
¼	¾	.03	9	81.8	100.0	128.6
½	½	.06	6	75.0	100.0	150.0
¾	¼	.09	3	60.0	100.0	300.0

When r equals k , P equals A validating the Miller and Modigliani proposition that dividends do not matter. A shortcut to a payout ratio is dividend yield times the P-E ratio or $(D/P)(P/E) = D/E = (1-b)$. A valuation using the price-earnings ratio follows from $P_0 = E_1(1-b)/(k-br)$ where $P-E = (1-b)/(k-br)$ and when multiplied by the expected earnings E_1 provides a stock valuation. In equilibrium when k equals r , the P-E ratio equals $1/k$.

Conclusion

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While obvious once the mathematics are examined, I repeatedly ask my students what is the holding period return to bonds and stocks and rarely get a correct response. Thus I'm repeatedly reminded that this exercise is well worth the review.

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