

**APPLYING THE FRACTAL MARKET HYPOTHESIS TO NIGERIA'S
EQUITY MARKET PERFORMANCE****Kenneth Uchenna Okafor**Professor of Financial Management, Department of Banking and Finance, Abia State University, Uturu,
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DOI: 10.5281/zenodo.19691113

Abstract

This study empirically tested the Fractal Market Hypothesis (FMH) in the Nigerian Stock Market (NSM) using Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) models. The FMH suggests that financial markets exhibit fractal characteristics, with price movements influenced by heterogeneous agents and long-term memory, as opposed to the random walk assumption of the Efficient Market Hypothesis (EMH). Daily stock price data from 2015 to 2024 were analyzed to assess volatility dynamics, fractal behavior, and asymmetric effects in the NSM. The findings revealed significant volatility clustering and asymmetric effects in stock returns, consistent with fractal behaviour. The EGARCH model demonstrated that negative shocks increase volatility more than positive shocks of equal magnitude, highlighting the leverage effect in the NSM. This supports the FMH, suggesting that stock prices are influenced by historical patterns, thus challenging market efficiency in its strictest form. These results have implications for both market participants and regulators. Investors may exploit the fractal characteristics and volatility patterns to enhance risk management and trading strategies. Policymakers should consider these dynamics to improve market transparency and stability. Future research could explore the impact of macroeconomic shocks on these fractal properties and extend this analysis to other emerging markets.

Keyword: Fractal Market Hypothesis, Nigerian Stock Market, EGARCH, Volatility Clustering, Asymmetric Effects, Stock Returns, Financial

Introduction

Al, 2022, Or Subash, 2012, Malkiel (2017). And some detractors have claimed that the Efficient Market Hypothesis (EMH) was mostly to blame for the crisis (Malkiel (2017). Efficiency of the stock market is constantly questionable and a topic of debate among Investors and academicians. An efficient stock market can be defined as the market where prices of the shares are adjusted according to the availability of new information. This means

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the current prices of shares reflect all the current information about the security. Efficient Market Hypothesis is the combination of some assumptions, like, negligible transaction costs, information either private or public is easily available to all investors, time horizon is same for all traders in the market or expectations of the investors are identical. Fama (1965) an efficient market is a market where investors are rational, they can maximize profit by predicting future market values of securities, where they can update their information which is freely available to all the participants. In other words, a market where the actual price of a security is a good estimate of its intrinsic value is an efficient market. He further irritated that the efficiency of a market as, the one in which the stock prices always fully reflect all the information available in the market. EMH claim that in an information efficient market, price changes cannot be predicted if the prices of stock reflect all the expectations of the investors and all the available information in the market (Bachelier, 1900, Cowles, 1933, Kendall, 1953, Samuelson, 1965, Fama, 1965 and Fama, 1970). According to the hypothesis, there is a correlation between stock prices and information because the stock prices are always formed according to the new information announced. In the EMH, markets scale from an asymmetric information environment, they argued that informational efficiency is achieved if securities prices display all the available information and this implies a random walk in stock prices i.e. each change in price is completely independent of previous price. Lo (1991), asserts that when stock returns have strong long-range dependence properties, they fail to portray a truly random walk process. Though, Peters (1994) opined that the presence of longterm memory in financial data implies that stock market do not immediately reflect new information as assumed by EMH. Proponents of this hypothesis have it that if long memory properties exist in asset returns then new information is not immediately portrayed in prices and events in the remote past are correlated with events in the foreseeable future (Mandelbrot 1971). They posited that it could be due to limits to arbitrage, irrationality and illiquidity among other factors. These arguments led to alternative models such as Fractal Market Hypothesis (FMH) to better explain the behaviour of stock returns. The Fractal market hypothesis (FMH) considers liquidity and investment horizon to be crucial for market Stability and that Prices changes in the market occur due to differences in interpretation of the same information by investors (Nyamute, Oloko, & Lishenga, 2017 and Peters, 1994). FMH emphasizes the need to have stable markets and a market is deemed to be stable when participants hold portfolios with different timelines given the same information. Investors who hold portfolios for longer period assume more risk as compared to those who hold for a shorter period (Thomas, 2002). For investors to realize market liquidity, FMH assumes investors attach different values to the securities to be traded and react differently given the same information. He further stated; to maintain balance in the market FMH proposes rules that govern trading and rest time between trades to encourage holding periods in portfolios and a clash in investing time horizons will cause crisis in the market. Adverse changes in information in the

market will result in market instability and shift in liquidity position to the worst. Therefore, stable markets equal high liquid stocks. Many researchers have worked and analyzed the efficiency of stock markets across the world and in Nigeria on whether the stock market is of EMH or FMH (Ejem, Ogbonna & Okpara, 2020; Ayankunle, Nwakanma & Torbira, 2021; Sapong, 2017; Kumar, Jayakumar & Kamaiah, 2017; Karp & Vuuren, 2019; Ghazani & Ebrahimi, 2019). The arguments have remained an unsolved puzzle and in order to buttress the view in line with the arguments the study became pertinent. Additionally, investors, regulators, and other players need clarity on the behavior of assets in the Nigerian stock market. Hence the thrust of the study is to examine the existence and effect of fractal market hypothesis and the behavior of stocks in the Nigerian Stock Market (the study is line with the study of Ayankunle, Nwakanma, and Torbira, 2021).

1.2 Statement of Problem

The Efficient Market Hypothesis (EMH), which posits that stock prices reflect all available information and follow a random walk, has been a cornerstone of financial theory (Fama, 1970). According to the EMH, it is impossible to consistently predict future stock prices based on past information, as price movements are assumed to be entirely random. However, this theory has come under increasing scrutiny, particularly in emerging markets like the Nigerian Stock Market (NSM), which are characterized by high volatility, low liquidity, and frequent price shocks. These market anomalies raise questions about the applicability of the EMH in such contexts. The Nigerian Stock Market exhibits significant volatility clustering, where periods of high volatility tend to be followed by more volatility, and vice versa. This behavior suggests that stock prices may not be entirely random and that past price movements could have predictive power for future market behavior. This challenges the core tenet of the EMH and suggests the need for alternative models to explain the behavior of stock returns in the NSM. One such alternative is the Fractal Market Hypothesis (FMH), which posits that markets are fractal in nature, exhibiting selfsimilarity and long memory across different time scales (Peters, 1994). Despite the importance of the NSM in Africa's financial landscape, there has been limited empirical research exploring whether the market exhibits fractal characteristics, such as long-term memory and volatility clustering, as predicted by the FMH. Furthermore, the presence of asymmetric volatility, where negative shocks to the market lead to disproportionately higher volatility than positive shocks (the leverage effect), has not been extensively studied in the context of the NSM. Traditional volatility models such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model may fail to capture the complexity of volatility dynamics in the NSM, necessitating the use of more advanced models like the Exponential GARCH (EGARCH) to accurately assess the market's behavior. The key problem addressed in this study is the gap in understanding whether the NSM exhibits fractal behavior and volatility clustering, as posited by the FMH, and how these dynamics influence

market efficiency, risk management, and investment strategies. This research aims to apply the EGARCH model to analyze stock return behavior in the NSM, providing empirical insights into the presence of fractal dynamics and asymmetry in volatility. Such insights are crucial for investors, policymakers, and academics seeking to understand and navigate the complexities of stock market behavior in emerging economies like Nigeria.

2. Literature Review

In this section, theoretical and empirical literatures are reviewed.

2.1 Efficient Market Hypothesis (EMH) Eugene Fama is credited with the first use of the term ‘efficient market’ in his paper titled: “Random Walks in Stock Market Prices,” (Fama, 1965) even though there is evidence that the concept was independently developed by himself and Paul Samuelson (Samuelson, 1965) from two separate research works (Durlauf & Blume, 2008). Fama (1965) defined efficient market as: “...a market where there are large numbers of rational profit maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants (Sarpong, 2017) Therefore, an efficient market is one that the prices of securities are reflective of the available information i.e. stock prices increase are spectacular on the day the news is made public but there is no further movement in prices after the date of the announcement, implying that prices incorporate the new information, which by the end of the trading day, includes the probable enormity of the takeover premium. (Patell & Wolfson, 1984; Busse & Green, 2002; Kane & Marcus, 2014; Hartman & Rodestedt, 2010; Kurov, Sancetta, Strasser & Wolfe, 2015) Furthermore, Fama (1965) categorizes market efficiency into three with respect to information available as weak- form, semi- strong, and strong form of efficiency. In weak form, all market prices bear all historical news (past information) on stock prices and arbitrage cannot be employed to beat the market. The semi-strong EMH suggests that the current asset prices incorporate all public information and investors cannot make above average risk adjusted returns. Finally, the strong form holds that prices reflect all information, both private and public (inside information) (Fama, 1970). The implication is that company insiders cannot consistently derive above average risk adjusted returns. Hence, EMH holds that investors make rational decisions while markets are efficient and investors cannot make above average returns (Lo, 1991). This shows that EMH invalidate anomalies in the market like January effect, Days of the Week and Calendar effect.

2.1.2 Fractal Market Hypothesis

The fractal market hypothesis (FMH) emerged in the 1990s from the work of Edgar Peters (1994). This was a follow up on his criticism of the EMH (Peters, 1991). He proposed the FMH, as a new method for modelling the deterministic characteristics and conflicting randomness of financial markets. A fractal is a figure that comprises

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parts that are similar to the main figure itself, therefore appearing approximately the same regardless of the scale observed. Scaling a fractal up or down by the same amount still results in self-similar patterns (Sarpong, 2017). FMH focused on the heterogeneity of investors with regard to their horizon. The Differences in investment horizons ensures that financial markets function in a stable manner. Generally, investors share the same degree of risk once with subsequent adjustments for the range of investment horizons. In fact, such “shared risk explains why the frequency distribution of returns look the same at different investment horizons” (Peters 1994), and is responsible for the fractal nature of financial markets. The market will become unstable if it loses its “fractal” nature. Market failures may occur when there are major uncertainties in long-term expectations. Wars, political crisis and natural disasters for example, can alter the fundamentals of financial markets. In such periods, long-term investors affected by such events, will adopt a short-term approach or totally avoid investing in the market. Shortening positions leads to a dry up of liquidity and subsequently a critical period where markets become highly volatile. Peters (1994) posits that so far as market participants with differing investment horizons are active in the market, a panic in one group can be easily contained by other horizons who will view such event as an opportunity to buy or sell. Conversely, if the market wholly assumes the same horizon, or a crucial segment of the market stay away from market activities, then the market will become unstable. In this situation, the nonexistence of liquidity eventually culminates in a panic. FMH asserts that price variations can still be regarded as random walks. However, this time the statistical distribution of the random walks is similar over different scales in time. This means that financial markets consist of multitudes of investors who have distinct investment horizons and whose information set is peculiar to their investment horizons. If this fractal structure is maintained, there should be stability in the financial markets. He further posited that the dominance of a specific investment horizon will lead to inefficiency ie clearance of buy and sell orders and this may climax in extreme events. Thus, FMH ascribes the existence of extreme events in financial markets to dominating investment horizons (Kristoufek, 2013). Nyakurukwa and Seetharam (2023) stated that there is no “agreed-upon mechanism” to determine the fair value of a stock. He irritated that, in the past few years, advances in technology have necessitated trading of large volumes of stock by spontaneous matching of buyers and sellers with distinct investing horizons. However, these technological advancements have only succeeded in matching buyers and sellers with no record of liquidity. The EMH simply subscribes to the notion of fair value regardless of whether adequate market liquidity exists or not. From the researcher’s view, the absence of adequate liquidity in the market may force participating agents to take any price, whether it is fair or not. Thus, liquidity plays a very crucial role in the FMH leading to the following propositions:

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- When investors from a wide range of investment horizons participate in the market, it is stable and traders have access to plenty of liquidity.
- Longer-term fundamental information predominates when investing horizons expand and price fluctuations may represent information exclusively relevant to a particular investment horizon.
- Market instability results from a market's entire investing horizon becoming uniformly smaller, for example, when long-term investors who provide short-term investors with liquidity, are not there to help stabilize the market (Nyakurukwa & Seetharam, 2023) In addition, while comparing EMH and FMH (Sarpong, 2017) posited that EMH emphasis is on fair asset prices and efficient markets while FHM is on Liquidity. Concerning market cycles and memory; EMH records that Past events have no effect on future prices as markets behave in a random manner while FMH, the path of the market is determined by past events thereby exhibiting deterministic order making short-term predictions possible. Also, for EMH Market has a single equilibrium and always in equilibrium with deviations that are highly infrequent and negligible deviations. FMH, there are different equilibrium for each investment horizon therefore the market cannot reach just a single equilibrium and its distribution is Fat tails and high peaks while that of EMH is normal Generally, FMH assumes several investment horizons with investors in the various horizons assuming the "counter-positions" thereby providing liquidity in the market all the time therefore ensuring stability of the market. The FMH therefore provides an alternative hypothesis on the behaviour of financial markets and provides an explanation to why financial markets fail leading to financial crises whereas the EMH simply brushes these crises aside simply as anomalies. (Sarpong, 2017)

2.3 Empirical Review

Karaömer (2022) investigated the market efficiency of emerging stock markets, (Mexico, Indonesia, Nigeria, and Turkey (MINT) stock markets based on the Fractal Market Hypothesis. the ARFIMA and ARFIMA-FIGARCH type models are used to analyze the MINT stock return series using daily frequency data of the MINT stock market indices from January 12, 2018, to January 12, 2022. Findings show that long memory is reported for the MINT stock returns. This implies that the returns of the MINT stock prices follow a predictable behavior that is consistent with the Fractal Market Hypothesis. The long memory in the volatility implies that the uncertainty or risk is an important factor in the formation of price movements in the MINT stock prices. Testing the existence of Long-term Memory in Stock Market Returns at Nairobi Securities Exchange Mutinda, Njeru and Mwangi (2022), explored the applicability of Fractal market hypothesis and Chaos theory in explaining market behaviour. The study employed a nonparametric test; classical rescaled range analysis to examine long term memory which is measured by the Hurst exponent developed by Hurst (1951). The stock market returns were considered using secondary data. The daily NSE-20 share index was collected for a period of eight years from January 2010 to

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December 2017. A longitudinal research design was employed for the research. The data was analyzed using E-views financial software. The results show that there is long term memory in stock market returns in NSE with an H-value of 0.7 from the rescaled range analysis. It is further observed that market returns are not normally distributed for the test of normality with a negative skewness of -0.067 and the autocorrelation denoted by P-value <0.05 showing the market does not follow a random which actually invalidates the efficient market hypothesis. This indicates that there exists a chance to predict market returns and make above market profits. The research recommends factoring in long term memory properties in investment decisions. Metescu (2022) in the paper Fractal Market Hypothesis vs. Efficient Market Hypothesis: Applying the R/S Analysis on The Romanian Capital Market. Determined the Hurst exponent for company ALRO S.A., for the period of time since listing, until 16/07/2021. Values obtained for Hurst exponent applied on the time series of ALRO S.A offer precise information about the presence of the memory effect inside the stock market. As a conclusion, as Alro S.A. is one of the most representative stocks traded at Bucharest Stock Exchange, Romanian capital market has evolved from a very low stability market to a more stable investment environment. Ehiedu and Obi (2022) studied Exchange in The Midst of Global Financial Crisis. All Share Index (ASI) was used to examine the Efficient Market Hypothesis and its effects on the global financial crisis. Monthly data from January 2, 2015 to December 20, 2020 (72 observations) and annual data from 1985 to 2020 (36 observations) were gathered from the CBN statistical bulletin. The Nigerian stock exchange was shown to be monthly form efficient for the yearly ASI throughout the study period, but weakly form efficient for the monthly ASI utilizing the unit root test, GARCH model, autocorrelation, and partial autocorrelation tests (1985-2020). The research on the monthly and annual ASI shows a substantial correlation between price series and their lag values, supporting the idea that price series in the Nigerian stock market do not follow a random walk for monthly ASI but do follow a random walk for yearly ASI. In other words, the results confirmed whether or not the mixed findings from the Nigerian Stock Exchange are effective in their weak form. Ayankunle, Nwakanma and Torbira, (2021) examined the tangibility and implication of fractal price hypothesis on the behaviour of the Nigerian, Egyptian, and the South African Stock Markets over the period of January 1995 to December 2020. The study employed secondary data gotten from the various sources; the Central Bank of Nigeria statistical bulletin, World Bank report, and the annual reports of Federal Reserve Economic Data. The employed data analysis techniques are descriptive statistics, the graphical trend of data, univariate data analysis such as the Correlograms Q-Statistics, the Breusch-Godfrey LM tests, autocorrelation test, stationarity test, and the Generalized Autoregressive Conditional Heteroskedasticity. The study observed significant fractal trends in the various countries. In terms of the relationship between fractal prices and liquidity/volume of transactions, the study observed a significant relationship between stock prices and

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liquidity position of the various countries. While no significant link between fractal prices and returns made by investors was uncovered by the study. Likewise, in an attempt to understand the fractal evidence in a capital Market Karp and Van Vuuren (2019) evaluated the time dependence of the supply of investors' liquidity to the market is explored for two developed market indices and one emerging market index. The study employed the Hurst exponent quantity to determine whether a fractal time series evolves by random walk, a persistent trend or mean reverts. Another quantity, the fractal dimension of a time series, provides an indicator for the onset of chaos when market participants behave in the same way and breach a given threshold. A relationship is found between these quantities: the larger the change in the fractal dimension before breaching, the larger the rally in the price index after the breach. In addition, breaches are found to occur principally during times when the market is trending. Ejem, Ogbonna and Okpara (2020) in their study, Efficient Market Hypotheses Controversy and Nigerian Stock Exchange Relations was done using All Share Index (ASI) with daily data from January 02, 2014 to May 20, 2019 (1333 observations) and annual data from 1985 to 2018 (34 observations) collected from the Nigeria Stock Market fact books. The study employed three analytical methods; the unit root test, GARCH Model and the Autocorrelation cum partial autocorrelation method for the assessment of weak form hypothesis on the daily and annual all share index in the Nigerian Stock market. The results indicated a significant relationship between the price series and their lagged values implying that stock price series do not follow a random walk process in Nigerian stock market. Thus, affirming that the Nigeria Stock Exchange is not efficient in weak form. Ngozi (2017) examined the effects of the Global Financial Crisis on the volume and value of shares traded on the floor of the Nigerian Stock Exchange, and to determine the effects of the Global Financial Crisis on the number of listed companies on the Nigerian Stock Exchange. Secondary data collected from the Nigerian Stock Exchange was employed. Kruskal-Wallis's test statistical tool was used. The result of the analysis found that global financial crisis has no significant effect on market capitalization in the Nigerian Stock Exchange. It was equally found that global financial crisis has no significant effect upon the value of shares traded on the floor of the Nigerian Stock Exchange. Rather the study revealed that there exists no significant relationship between the Global Financial Crisis, and the volume of shares traded on the floor of the Nigerian Stock Exchange and that there exists no significant relationship between the Global Financial Crisis and number of listed companies in the Nigerian Stock Exchange. Spong (2017) investigates and confirms the low volatility anomaly on the Johannesburg Stock Exchange (JSE) using the risk-adjusted return measure of the Sharpe ratio. The study applied the Fractal Market Hypothesis (FMH) formalized within the framework of Chaos Theory, to explain the existence of the low volatility anomaly on the JSE. Building upon the Fractal Market Hypothesis to provide evidence on the behaviour of returns time series of selected indices of the JSE, the BDS test is applied to test for non-random chaotic

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dynamics and further applies the rescaled range analysis to ascertain mean reversion, persistence or randomness on the JSE. The BDS test confirms that all the indices considered in this study are not independent and identically distributed. The study further analyses the three fundamentals of the FMH namely, the impact of information, the role of liquidity and time horizon on the top 40 and small cap indices. Information is not uniformly distributed among the two indices as the FTSE/JSE Top 40 index receives more publications from sources such as newspapers, online publications and journals as well as JSE issued news and historical company news. The study finds that domestic equity fund managers in South Africa hold in their portfolios, a disproportionately greater percentage of FTSE/JSE Top 40 companies relative to other companies on the JSE and concludes that these managers contribute to the low volatility anomaly on the JSE. The study further concludes that in line with the FMH, lack of information and the illiquidity of the FTSE/JSE Small Cap attracts long-term investors who become the dominant class of investors on the index and are compensated for taking on the risk of illiquidity in the form of illiquidity premium and low volatility. The highly liquid FTSE/JSE Top 40, which has relatively high availability of information on the other hand attracts different classes of investors with differing horizons who take opposite sides of each trade as different classes of investors interpret the same set of information differently. The high liquidity and information lead to high volatility as investors continually adjust their holdings with the emergence of new information. The high volatility and subsequent underperformance of the FTSE/JSE Top 40 therefore is a cost of efficiency and liquidity (liquidity discount). Employing a non-parametric test; classical rescaled range analysis to examine long term memory which is measured by the Hurst exponent developed by Hurst (1951) Mutinda (2018) studied the applicability of Fractal market hypothesis and Chaos theory in explaining market behaviour In Nairobi. The stock market returns were considered using secondary data; NSE-20 share index was collected for a period of eight years from January 2010 to December 2017. The results show that there is long term memory in stock market returns in NSE with an H-value of 0.7 from the rescaled range analysis. It was further observed that market returns are not normally distributed for the test of normality with a negative skewness of -0.067 and the autocorrelation denoted by P-value <0.05 showing the market does not follow a random which actually invalidates the efficient market hypothesis. This indicates that there exists a chance to predict market returns and make above market profits. The research recommends factoring in long term memory properties in investment decisions. Kumar, Jayakumar, and Kamaiah (2017), studied nine Asian forex markets of China, India, Hongkong, Japan, South Korea, Singapore, Sri Lanka, Taiwan and Thailand for the period from 05-01-1994 to 30-06-2017 to test the Fractal Markets Hypothesis (FMH) proposition which states that a financial market can plunge into crisis when a particular trading time horizon gains prominence over others. A wavelet-based method was used to capture the activities in different timescales also, the bilateral daily exchange rate of

the corresponding currencies against the U.S Dollar were used. The time period covers two major crises and they are the 1997-8 East Asian currency crisis and the 2008 global financial crisis. The study captured both the events and from the wavelet spectra it is evident that the crisis period distinguishes itself with increased activity by the short-term traders as proposed by the FMH. The study found that the 1997-8 crisis affected not only the East Asian markets but also the other forex markets as well. Nwosa and Oseni (2011) in their paper Efficient Market Hypothesis and Nigerian Stock Market examined the weak-form efficient market hypothesis in the Nigerian stock market, using a sample data spanning the period 1986 and 2010. The study adopted a serial auto-correlation and regression method of analysis. The variables used in the study were tested for stationarity using the Augmented Dickey Fuller and Philip Perron test. The result showed that the variables are stationary at first differencing. The result of the serial auto-correlation and regression analysis both revealed that the Nigeria stock market is informational inefficient, that is stock price does not exhibit random walk. The study recommended that to enhance informational efficiency of the Nigerian stock exchange. Okpara (2010) investigated Stock market prices and the random walk hypothesis: Further evidence from Nigeria. The objective was to determine whether or not security prices fully reflect historical price or return information. Investigations were carried out with data quoted stock prices of 121 randomly selected securities listed throughout the period January, 1984 to December 2006 in the Nigerian stock market. The run test and the correlogram/partial autocorrelation function (Ljung - Box Q - statistics) as alternate forms of the research instrument were used to test the data. The results of the three alternate tests revealed that the Nigerian stock market is efficient in the weak form and therefore follows a random walk process. Thus, the opportunity of making excess returns in the market is ruled out.

3. Materials and Methods

This study employs an EGARCH model to analyze the fractal behavior and volatility dynamics of stocks in the Nigeria Stock Market (NSM). The EGARCH model captures both the magnitude and sign of past innovations, allowing for asymmetric effects in volatility, which are consistent with the leverage effect observed in financial markets (Engle & Patton, 2001).

3.1 Data Collection

The study uses daily closing prices of a sample of stocks listed on the Nigerian Exchange Group (NGX) from January 2015 to August 2024. The sample includes stocks from various sectors to ensure a comprehensive analysis.

3.2 EGARCH Model Specification

The EGARCH (p, q) model, proposed by Nelson (1991), is used to model the conditional volatility of stock returns. The model is specified as:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{\mu_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left(\frac{|\mu_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right)$$

Where, $\omega, \beta, \alpha, \gamma$ are constant parameters, $\text{Log}(\sigma_t^2)$ = the one period ahead volatility forecast, ω = the mean level, β = persistence parameter, α = volatility clustering coefficient, $\text{Log}(\sigma_{t-1}^2)$ = the past variance, γ = the leverage effect. The above model ensures that even when the parameters are negative, σ_t^2 will be positive and the asymmetry or the leverage effect measure, γ , will be negative even when the relationship between volatility and log returns is negative. The EGARCH is symmetric when $\alpha = 0$, when $\gamma < 0$ then positive shocks (good news) generate less volatility than bad news (negative shocks); in other way round, bad news or negative shocks magnify more volatility than good news or positive shock of the same magnitude. When $\gamma > 0$, it implies that positive innovations or shocks are more destabilizing than negative innovations or shocks (Black, 1976; Christie, 1982). In other words, negative value of γ is called the 'sign effect'. The choice of EGARCH framework is to accommodate examination of conditional variance (volatility), asymmetric effect and volatility persistence. The α ' parameter represents the symmetric effect of the model, if α is positive, then the conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller), hence magnitude effect'. The GARCH effect β measures the persistence in conditional volatility. When β is relatively large, then volatility takes a long time to fizzle out or decay or die out following mayhem in the market or economy in general. Succinctly, the EGARCH model has a good number of advantages over the normal GARCH specification. First, since the $\log(\sigma_t^2)$ is modeled, then even the parameters σ_t^2 will be positive. There is thus no need to artificially impose non-negativity constraints on the model parameters. Second, asymmetries are allowed under the EGARCH formulation, since if the relationship between volatility and returns is negative, γ , will be negative (Brooks, 2008). The model parameters are estimated using maximum likelihood estimation (MLE). The Akaike Information Criterion (AIC) is used to select the optimal model order (p, q).

3.3 Determination of Expected Returns

The price data is converted into compound returns by taking logarithms:

$R_t = \ln(p_t/p_{t-1})$, Where R_t , is the current market returns, p_t is the current market index price, p_{t-1} is the previous market index price.

4. Data Analysis

4.1. Test of ARCH Effect

Table 4.1 below shows that the F-version and the LM-statistics are significant, indicating presence of ARCH effects in the returns

Table 4.1 ARCH Effect Test

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ARCH Test	NIG
F-statistic	0.0000
Obs*R-squared	0.0000

4.2. EGARCH Model Results

Table 4. 2 Estimation of models using EGARCH

Parameter Estimates	NIG
Mean Eqn	
b_2	3.367944 *0.4528
μ	0.204403 *0.0000
ω	-0.000266 *0.4411
Variance Eqn	
ω	-1.693731 *0.0000
α	0.375256 *0.0000
γ	-0.063860 *0.0000
β	0.849384 *0.0000
Log likelihood	6482.626
Dw Stat	1.830290
AIC	-6.665595
SIC	-6.645521
P-value of ARCH LM Test	*0.9398

Probability values

Here, the researchers employed EGARCH as shown on table 4.2. The result of the analysis revealed that the leverage effect or asymmetry parameter γ are negative and significant, suggesting presence of leverage effects, implying that bad or negative news cause more volatility than good or positive news of the same magnitude. Furthermore, the persistent parameter β are positive and significant, also are relatively large indicating that

volatility is persistent, confirming that volatility takes a long time to die following the crisis. Magnitude effect (α) (volatility clustering) coefficient of EGARCH is positive and significant. That means the conditional volatility will rise or fall when the absolute value of the standardized residual is larger (smaller). Table 4.2 below also found that the ARCH- LM tests for the serial correlations were insignificant at 5% critical level for the study, suggesting that the asymmetry models are sufficient in modeling the serial correlation structure in the conditional mean and variance. This indicates there is no further ARCH effect in the estimated ARCH-GARCH models, as well as suggesting that the models are correctly specified. The AIC and SIC were found to maintain small criterion value for all the variants of ARCH for the study, affirming the suitability of the models, hence are best fit models.

4.1 Discussion

The evidence of fractal behavior and volatility clustering in the NSM supports the FMH, suggesting that stock prices are not entirely random but are influenced by historical patterns and investor behavior. The presence of asymmetric effects in volatility further indicates that negative shocks have a more pronounced impact on volatility than positive shocks, consistent with the leverage effect observed in other markets

Fractal Characteristics and Volatility Clustering

The EGARCH model results show evidence of volatility clustering, consistent with the fractal characteristics predicted by the FMH. The long memory properties in volatility suggest that stock returns are influenced by past information, challenging the random walk assumption of the EMH. The empirical analysis reveals that the NSM exhibits fractal properties and volatility clustering, as suggested by the FMH. The presence of asymmetric effects in the volatility of stock returns further supports the idea that market dynamics are driven by heterogeneous agents with varying investment horizons. The findings indicate that the NSM is not fully efficient in the EMH sense, as past information can influence future price movements. This has implications for investors, who may exploit predictable patterns in volatility to enhance their trading strategies.

Comparison with Other Markets

The findings align with studies in other emerging and developed markets, which also show evidence of fractal behavior and volatility clustering (Zunino et al., 2008). However, the degree of fractality and volatility asymmetry may differ due to factors such as market maturity, investor composition, and regulatory environment.

Policy Implications

The presence of fractal properties and volatility clustering in the NSM suggests that regulators should consider these findings when designing policies to enhance market stability and transparency. For instance, measures to improve market liquidity and reduce information asymmetry may help mitigate volatility clustering and improve market efficiency.

Conclusion and Recommendations

5.1 Conclusion

This study provides empirical evidence supporting the Fractal Market Hypothesis (FMH) in the Nigerian Stock Market (NSM) using EGARCH models to analyze stock return behavior. The results reveal significant fractal characteristics, particularly in the form of volatility clustering and asymmetric effects, where negative shocks lead to greater increases in volatility compared to positive shocks of the same magnitude. These findings challenge the Efficient Market Hypothesis (EMH) by suggesting that stock prices in the NSM do not follow a pure random walk and are influenced by historical price movements, indicating the presence of long memory and predictability. The presence of volatility clustering indicates that shocks to the market have a persistent effect, implying that the market is not fully efficient and that investors can potentially exploit these patterns. Additionally, the leverage effect, where negative market shocks result in disproportionately higher volatility, highlights the risk dynamics present in the NSM, making it essential for market participants to account for these fluctuations in their trading and risk management strategies.

5.2 Recommendations for Investors:

- i. **Exploitation of Fractal Patterns:** Investors should incorporate the presence of long memory and volatility clustering in their trading strategies. By recognizing periods of high volatility or market trends, investors can make more informed decisions and potentially achieve better returns.
- ii. **Risk Management:** Given the leverage effect observed in the study, investors should use advanced risk management tools such as stop-loss mechanisms, portfolio diversification, and volatility-based trading strategies to minimize the impact of sudden negative market shocks.

For Policymakers and Regulators:

- i. **Enhancing Market Stability:** Policymakers should design regulatory frameworks that mitigate the effects of volatility clustering by improving market liquidity, transparency, and accessibility. This could help reduce the persistence of shocks and create a more stable trading environment.
- ii. **Information Dissemination:** The Nigerian Stock Exchange (NSE) should enhance the dissemination of timely and relevant information to all market participants. This could improve market efficiency by reducing information asymmetry and allowing investors to react to market developments more quickly.
- iii. **Investor Education:** Regulators should focus on educating investors about market dynamics, particularly regarding fractal market behavior and the risks of volatility clustering. Such education could help investors make more rational decisions in times of market turbulence.

Suggestions for further Research:

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- i. **Exploring External Shocks:** Future studies should examine how external macroeconomic factors such as oil prices, exchange rates, and interest rates affect the fractal properties and volatility behavior of the NSM.
- ii. **Comparative Market Studies:** Researchers could extend this study to other emerging and developed markets to compare the extent of fractal behavior and volatility clustering across different market environments.
- iii. **High-Frequency Data Analysis:** Future research could utilize high-frequency data to provide a more detailed examination of intra-day stock price movements and further validate the presence of fractal dynamics in the NSM.

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